Applied Acoustics 130 (2018) 99–106

Contents lists available at ScienceDirect

Applied Acoustics

journal homepage: www.elsevier.com/locate/apacoust

Aeroacoustic scattering of rotating sources using a frequency-domain acoustic pressure gradient formulation

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ARTICLE INFO

Article history:
Received 17 July 2017
Received in revised form 7 September 2017
Accepted 13 September 2017

Keywords:
Computational aeroacoustics
Acoustic scattering
Rotor noise
Equivalent sources
Acoustic pressure gradient

ABSTRACT

In this paper, a frequency-domain methodology for acoustic scattering prediction from rotating sources is suggested. Frequency-domain acoustic analogy formulations are used for incident field prediction of acoustic pressure and the acoustic pressure gradient. The latter is required for evaluation of the hardwall boundary condition on the scattering surface. The boundary value problem is solved by the equivalent source method. The overall scattering approach is validated by an analytical case of scattering by a monopole point source rotating around a rigid sphere and by a hovering non-lifting helicopter rotor operating above an infinite flat plate. The implemented acoustic pressure gradient formulation can be coupled to any boundary method which requires a solution of the incident field on the hardwall scattering surface, for acoustic scattering computation from rotating sources. This scattering methodology is applicable to acoustic scattering prediction from rotating machines, allowing computation of specific tones of interest, thus being particularly efficient.

1. Introduction

Noise generated from rotors and turbomachinery comprises of incident and scattered acoustic components. When realistic configurations of turbomachinery or installed aircraft engines are concerned, the reflected and scattered sound waves may be of greater amplitude than the incident acoustic field [1]. Therefore, accurate and efficient aeroacoustic scattering prediction in rotating machines is necessary for industrial applications.

Computation of incident acoustics generally relies on hybrid computational aeroacoustic (CAA) methods which allow efficient decoupling of flow and noise. On the other hand, aeroacoustic scattering prediction is mostly based on boundary methods (such as the boundary element method [2,3] and the equivalent source method (ESM) [4,5], among others) which rely on evaluation of the hardwall boundary condition on the scattering surface. These methods require computation of the acoustic velocity or acoustic pressure gradient components on the scattering surface, to satisfy this boundary condition.

The hardwall boundary condition was initially evaluated by relating the acoustic velocity to the acoustic pressure gradient through the linearized momentum equation. This allowed numerical computation of the acoustic pressure gradient on the scattering surface [6]. However, this procedure can be costly for realistic configurations. Lee et al. [7] obviated the need for such a computation by proposing for the first time an analytical formulation for acoustic pressure gradient prediction in the time domain. This formulation was coupled to the time-domain ESM [8] and applied to acoustic scattering of rotorcraft noise [9]. Recently, Mao et al. [10] proposed a scattering methodology for rotating sources in the frequency domain, where boundary condition evaluation relies on a frequency-domain solution of acoustic velocity [11] and scattered noise is computed by the original ESM [4].

Ghorbaniasl et al. [12] have suggested a frequency-domain solution of the Ffowcs-Williams and Hawkings equation [13] (FW-H) for analytical acoustic pressure gradient prediction. The said formulations have been derived in a moving medium and have been applied for scattering predictions in the presence of a uniform flow [14]. Since these formulae can include the effect of flow and angle of incidence on the acoustic simulations, they display significant potential for acoustic scattering of rotating acoustic sources, such as rotors.

Scope of the present paper is to demonstrate the implementation of the acoustic pressure gradient formulations suggested by Ghorbaniasl et al. [12] in a frequency-domain scattering methodology

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https://doi.org/10.1016/j.apacoust.2017.09.010
0003-682X/

for rotating noise sources. The analytical formulations for acoustic pressure gradient prediction can be coupled to any scattering methodology which requires evaluation of the hardwall boundary condition. Here, the scattered field solution is obtained by the ESM [4], also in the frequency-domain, since it is efficient and relatively simple to implement.

The validity of the numerical methodology for prediction of rotating source noise will be initially assessed against the analytical test case of a rotating monopole noise source scattered by a sphere [10]. Application of the overall scattering approach will be realized on a hovering helicopter rotor, operating above a flat scattering surface. The computed incident and scattered acoustic fields of the rotor will be validated and discussed.

2. Theoretical background

The suggested methodology for acoustic scattering prediction is based on a moving-medium frequency-domain solution of the FW-H equation for prediction of the acoustic pressure and acoustic pressure gradient [12]. The former is required for incident noise prediction, whereas the latter allows the evaluation of the boundary condition on the scattering surface. The resulting boundary value problem is then solved by the ESM [4]. The theoretical background of the numerical methodology is provided in this section.

2.1. Analytical acoustic pressure gradient prediction in the frequency domain

The convected FW-H equation [13] for a permeable data surface in the time domain is given in [15] by

$$\left[\frac{1}{c_0^2}\frac{D^2}{Dt^2}-\nabla^2\right]\{p'(\boldsymbol{x},t)H(f)\}=\frac{D}{Dt}[Q\delta(f)]-\frac{\partial}{\partial x_\alpha}[F_\alpha\delta(f)] \tag{1}$$

where p' is the acoustic pressure, c_0 is the speed of sound and t is the observer time. The source terms are defined as follows:

$$\begin{aligned}F_\alpha&=L_\alpha-QU_{\infty\alpha}\\ L_\alpha&=\rho u_\alpha[u_n-(v_n-U_{\infty n})]+P_{\alpha\beta}\hat{n}_\beta\\ Q&=\rho[u_n-(v_n-U_{\infty n})]+\rho_0(v_n-U_{\infty n})\end{aligned} \tag{2}$$

Here, $v_n=v_\alpha\hat{n}_\alpha$ with $\hat{n}_\alpha=\partial f/\partial x_\alpha$ being the local unit outer normal and v_α the data surface velocity. Furthermore, $D/Dt=\partial/\partial t+U_{\infty\alpha}\partial/\partial x_\alpha$, where $U_{\infty\alpha}$ denotes the constant uniform velocity components. $u_n=u_\alpha\hat{n}_\alpha$, with u_α being the local flow velocity components and $U_{\infty n}=U_{\infty\alpha}\hat{n}_\alpha$. Variable ρ is the local fluid density, whereas ρ_0 denotes the fluid density at rest. In Eq. (2), $P_{\alpha\beta}$ represents the compressive stress tensor, where the flow quantities are given in the coordinates fixed to the medium at rest. Additionally, $\delta(f)$ is the Dirac delta function and $H(f)$ is the Heaviside function of the data surface, $f(\boldsymbol{x},t)$, so that

$$\delta(f)=\frac{\partial H(f)}{\partial f},\ \frac{\partial H(f)}{\partial x_\alpha}=\delta(f)\hat{n}_\alpha,\frac{\partial H(f)}{\partial t}=-v_n\delta(f) \tag{3}$$

Due to the Heaviside function, Eq. (1) is valid in the entire domain outside the data surface.

In order to derive a solution for Eq. (1), the Green function can be expressed as [15]

$$G(\boldsymbol{x},t;\boldsymbol{y},\tau)=\frac{1}{2\pi}\int_\omega\frac{e^{-ikR}}{4\pi R^*}e^{i\omega(t-\tau)}d\omega \tag{4}$$

where τ is the source time, $i=\sqrt{-1}$, and $k=\omega/c_0$ is the acoustic wavenumber, with ω being the angular frequency. The phase radius R denotes the time delay between emission and reception, whereas R^* is the amplitude radius representing amplitude decay of the acoustic signal. R and R^* are written as [15]:

$$R^*=\frac{r}{\gamma}\sqrt{1+\gamma^2M_{\infty r}^2},\quad R=\gamma^2(R^*-rM_{\infty r}) \tag{5}$$

Here, $r=|\mathbf{x}-\mathbf{y}(\tau)|$ is the distance between the source position $\boldsymbol{y}$ and the observer position $\boldsymbol{x}$. Moreover, $M_{\infty r}=M_{\infty\alpha}\hat{r}_\alpha$ with $\hat{r}_\alpha=r_\alpha/r$ and $\gamma=1/\sqrt{1-|\boldsymbol{M}_\infty|^2}$. The uniform constant flow Mach number is defined as $\boldsymbol{M}_\infty=\boldsymbol{U}_\infty/c_0$.

Using the Green's function provided in Eq. (4), a solution can be finally derived in the frequency domain as [12]:

$$\tilde{p}'(\mathbf{x},\omega)=\tilde{p}'_1(\mathbf{x},\omega)+\tilde{p}'_2(\mathbf{x},\omega) \tag{6}$$

In the present paper, a frequency-domain formulation for analytical prediction of the acoustic pressure and acoustic pressure gradient for moving data surfaces is used, as suggested by Ghorbaniasl et al. in [12]

$$\begin{aligned}4\pi\tilde{p}'_1(\mathbf{x},\omega)&=\int_S\int_{-\infty}^{T_{int}}\left[i\omega\frac{Q(\boldsymbol{\eta},\tau)}{R^*}\right]e^{-ikR}e^{-i\omega\tau}d\tau dS\\ 4\pi\tilde{p}'_2(\mathbf{x},\omega)&=\int_S\int_{-\infty}^{T_{int}}\left[\frac{ikF_R(\boldsymbol{\eta},\tau)}{R^*}+\frac{F_{R^*}(\boldsymbol{\eta},\tau)}{R^{*2}}\right]e^{-ikR}e^{-i\omega\tau}d\tau dS\end{aligned} \tag{7}$$

and

$$4\pi\frac{\partial\tilde{p}'_1(\boldsymbol{x},\omega)}{\partial x_\alpha}=\int_S\int_{-\infty}^{T_{int}}-i\omega\left[\frac{ik\hat{R}_\alpha}{R^*}+\frac{\hat{R}^*_\alpha}{R^{*2}}\right]Qe^{-ikR}e^{-i\omega\tau}d\tau dS$$

$$\begin{aligned}&4\pi\frac{\partial\tilde{p}'_2(\boldsymbol{x},\omega)}{\partial x_\alpha}\\ &=\int_S\int_{-\infty}^{T_{int}}\left[\frac{k^2F_R\hat{R}_\alpha}{R^*}+ik\frac{F_\alpha-(\gamma^2F_{R^*}+F_R)\hat{R}^*_\alpha-F_{R^*}\hat{R}_\alpha+\gamma^2M_{\infty F}M_{\infty\alpha}}{R^{*2}}\right.\\ &\left.+\frac{F_\alpha-3\gamma^2F_{R^*}\hat{R}^*_\alpha+\gamma^2M_{\infty F}M_{\infty\alpha}}{\gamma^2R^{*3}}\right]e^{-ikR}e^{-i\omega\tau}d\tau dS\end{aligned} \tag{8}$$

where T_{int} is the integration time for the frequency of interest, $F_R=F_\beta\hat{R}_\beta$, $F_{R^*}=F_\beta\hat{R}^*_\beta$, and $M_{\infty F}=M_{\infty\beta}F_\beta$. As also derived in [15], one has:

$$\begin{aligned}\hat{R}^*_\beta&=\frac{\partial R^*}{\partial x_\beta}=\frac{r}{\gamma^2R^*}(\hat{r}_\beta+\gamma^2M_{\infty r}M_{\infty\beta})\\ \hat{R}_\beta&=\frac{\partial R}{\partial x_\beta}=\frac{r}{R^*}(\hat{r}_\beta+\gamma^2M_{\infty r}M_{\infty\beta})-\gamma^2M_{\infty\beta}\end{aligned} \tag{9}$$

Equations (7) and (8) enable calculation of the acoustic pressure and acoustic pressure gradient, respectively, for a rotating data surface. Equation (7) is used in the present study for prediction of the incident acoustic field, whereas Eq. (8) provides the acoustic pressure gradient components on the scattering surface, which are necessary for evaluation of the hardwall boundary condition by the scattering solver.

2.2. Acoustic scattering by the equivalent source method

The basic principle of the ESM is to replace the boundary value problem with a collection of point monopoles inside the scattering body. The strength of the equivalent sources is determined by satisfying the boundary condition at a finite number of nodes on the scattering surface. These equivalent sources in turn propagate the scattered acoustic field.

As shown by Dunn and Tinetti [4], if a small disturbance of one frequency component (with frequency ω) is considered, the acoustic boundary value problem can be expressed as

$$\frac{\partial p'}{\partial n}=0 \tag{10}$$

where p represents the acoustic pressure and a prime denotes perturbation from the mean.