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Microstructures for lowering the quarter wavelength resonance frequency of a hard-backed rigid-porous layer

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ABSTRACT

The frequency of the quarter wavelength resonance in the sound absorption spectra due to a thin hard-backed rigid-porous layer can be influenced by the design of its microstructure as well as its thickness. Microstructures considered include parallel arrays of identical cylindrical, slit-like or rectangular pores with deep sub-wavelength cross sections inclined to the surface normal, cylindrical annular pores, slits with log-normal width distributions, slits with cross sections that vary in a sinusoidal manner and slits with two distinct widths (dual porosity). Formulae that predict the bulk acoustical properties due to these microstructures are presented and used to explore the extent to which specific microstructures could be used separately or in combination to improve low-frequency absorption. Predicted normal incidence absorption coefficient spectra are compared using microstructural dimensions that would be feasible for 3D printing. The most effective microstructures are predicted to be slits with sinusoidally-varying widths, or with two distinct widths, inclined to the surface normal at 70°. The quarter wavelength layer resonances predicted in absorption coefficient spectra using these microstructures are comparable with those predicted for layers of the same thickness and bulk porosity having cylindrical pores with dead-end branches.

1. Introduction

Thin lightweight sound absorbing layers are required for several applications, in vehicles for example. A common problem for thin porous absorbers, which depend on viscous friction and thermal exchanges in pores, is that the low frequency absorption is poor. Low-frequency tuned absorption can be provided by Helmholtz resonators but these, along with many metamaterial alternatives to traditional porous absorbers, may be too bulky. Sound absorbing metallic foams, made by fusing metal spheres together and then perforating the fused sphere assemblies, are of interest because of their rigidity, lightness, thermal conductivity and their low production cost compared to open-cell metallic foams [1]. A hard-backed 0.0264 m thick perforated metallic foam has a narrow quarter wavelength layer resonance with a peak normal incidence absorption coefficient of 1 at 1 kHz and has estimated bulk parameters of porosity 0.283, flow resistivity 8.4 kPa s m^{-2} and tortuosity 5.54 [1].

By using 3D-printing technology, it is possible to manufacture rigid-porous sound absorbing materials with precisely specified microstructures. The average measured absorption coefficient spectrum of a 44.4 mm diameter hard-backed 0.03 m thick 3D printed cylindrical sample with overall porosity 0.234, incorporating cylindrical main pores of diameter 3.2 mm normal to the surface and cylindrical dead-end pores of diameter 1.3 mm normal to the main pores, has a broad quarter-wavelength layer resonance peak with magnitude 0.9 near 2.2 kHz [2]. A proposed 0.03 m thick layer with such a dead-end pore microstructure is predicted to result in a narrow absorption coefficient peak of magnitude 0.85 at 1.25 kHz. Another dead-end-pore microstructure design for a 0.035 m thick layer is predicted to give a narrow absorption coefficient peak with magnitude 0.65 at 0.65 kHz [2]. It has been found that hard-backed layers with cylindrical annular pores yield higher absorption than layers with the same thickness but with open cylindrical pores [3]. The measured absorption coefficient peak magnitude of 0.55 near 3.5 kHz for a 0.025 m thick sample containing 2 mm diameter cylindrical pores and with porosity 0.26 is increased to 0.95 near 3 kHz by inserting cylindrical rods into the open pores thereby creating annular pores of width 0.25 mm and reducing the bulk porosity to 0.11 [3].

Another simple microstructure is an array of uniform identical parallel-sided narrow channels or slits parallel to the surface normal. Two ways of improving the low-frequency absorption of thin porous layers containing parallel uniform slits have been investigated. One involves varying the width of the slits in a sinusoidal manner [4]. The other is to introduce two arrays of slits at right angles to each other, one with relatively large and the other with relatively small widths [5].

Analytical models for the acoustical properties of cylindrical,

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annular and slit microstructures are outlined and extended to allow for inclination of the pores to the surface normal since this increases the bulk tortuosity and thereby lowers the quarter wavelength resonance frequency. The extended models are used to investigate the extent to which each form of microstructure either separately or in combination could be used to improve low-frequency absorption of a thin hard-backed layer. Numerical comparisons use microstructure dimensions on the order of those of metallic foams [1] or of those that have been used elsewhere [2–5]. The chosen dimensions are within the resolution of commonly-available 3D-printers (0.1 mm at present).

2. Models for uniform identical pores

2.1. Parallel identical uniform pores

If the complex density in a (single) uniform pore of arbitrary shape is written

$$\rho(\omega) = \rho_0/H(\lambda) \tag{1}$$

where ω is the angular frequency and λ is a dimensionless parameter, then the complex compressibility [6–8] is

$$C(\omega) = (\gamma P_0)^{-1}[\gamma-(\gamma-1)H(\lambda\sqrt{(N_{PR})})] \tag{2}$$

where $(\gamma P_0)^{-1} = (\rho_0 c_0{}^2)^{-1}$ is the adiabatic compressibility of air, P_0 is atmospheric pressure and γ is the ratio of specific heats.

Expressions for $H(\lambda)$ and λ corresponding to various pore cross sections are listed in Table 1 in which $\nu = \mu/\rho_0$, where μ is the dynamic viscosity coefficient, ρ_0 is the (equilibrium) density of air, c_0 is the adiabatic sound speed in air and time dependence $\exp(-i\omega t)$ is understood.

The dimensionless parameter for an arbitrarily-shaped pore, λ can be related to the (steady) flow resistivity (R_s) of the bulk material of porosity Ω through the Kozeny-Carman formula [9],

$$R_S = \frac{2\mu T s_0}{\Omega r_h^2}, \tag{3}$$

where the hydraulic radius, $r_h = \frac{'\text{wetted}'\text{area}}{\text{perimeter}}$, s_0 is a steady flow shape factor and T is tortuosity, defined as the square of the increase in path length per unit thickness of material due to deviations of the steady-flow path from a straight line, and Ω is porosity.

Expressions for r_h and s_0 are listed in Table 2.

If the pores are inclined at angle θ to the surface normal, then [10]

$$T(\theta) = 1/\cos^2(\theta) \tag{4}$$

The complex density ($\rho_b(\omega)$) and complex compressibility ($C_b(\omega)$) for the bulk material are calculated from those for an individual pore using Eqs. (5a,b):

$$\rho_b(\omega) = (T/\Omega)\rho(\omega), C_b(\omega) = \Omega C(\omega) \tag{5a,b}$$

The bulk propagation constant ($k(\omega)$) and relative characteristic impedance ($Z_c(\omega)$) of the porous material consisting parallel slits of width $2b$ and edge-to-edge separation $b(1-\Omega)/\Omega$ may be calculated from Eqs. (6a,b).

$$k(\omega) = \omega[\rho_b(\omega)C_b(\omega)]^{0.5}, Z_c(\omega) = (\rho_0 c_0^2)^{-1}[\rho_b(\omega)/C_b(\omega)]^{0.5} \tag{6a,b}$$

Table 2
Hydraulic radius and steady flow shape factors for four pore shapes [9].

shape	r_h	s_0
slit, width $2b$	b	1.5
cylinder, radius a	$\frac{a}{2}$	1
equilateral triangle, side d	$\frac{d}{4\sqrt{3}}$	$\frac{5}{6}$
square, side $2a$	$\frac{a}{2}$	0.89

The surface impedance of a hard-backed porous layer of thickness d is,

$$Z(d) = Z_c \coth(-ikd) \tag{7}$$

The plane wave reflection coefficient, $R(d)$, and normal incidence absorption coefficient, $\alpha(d)$, for a hard -backed porous layer are given by Eqs. (8a,b):

$$R(d) = \frac{\rho_0 c_0 - Z(d)}{\rho_0 c_0 + Z(d)}, \quad \alpha(d) = 1-|(R(d))^2| \tag{8a,b}$$

respectively.

Using the values in Table 2 with Eq. (3), if a medium contains slits of width $2b$, $r_h = b$, $s_0 = 1.5$, and $R_s = 3\mu T/\Omega b^2$. If a medium contains cylindrical pores of radius a, $r_h = a/2$, $s_0 = 1$, and $R_c = 8\mu T/\Omega a^2$. For a given porosity and tortuosity, a layer with cylindrical pores of radius a will have 8/3 times the flow resistivity of a medium containing slits with semi-width a. A medium containing square pores of side $2a$ will have 0.89 of the flow resistivity of one containing cylindrical pores with radius a.

2.2. Cylindrical annular pores

Exact equations for a cylindrical annular pore involve cylindrical Bessel functions, as do those for an open cylindrical pore [3]. However, it has been shown [3] that, a semi-phenomenological model for an arbitrary pore microstructure, the Johnson-Champoux-Allard-Lafarge (JCAL) model [11,12], enables sufficiently accurate predictions of the acoustical properties of a layer containing cylindrical annular pores normal to the surface. In general, the JCAL model introduces viscous and thermal permeabilities, viscous and thermal characteristic lengths, porosity and tortuosity. For cylindrical annular pores, the viscous and thermal characteristic lengths are equal, as are the viscous and thermal permeabilities. Calculations for annular pores with outer radius R, ratio r of outer-to-inner radii and centre-to-centre spacing L are made using Eqs. (9) and (10).

$$\rho_b(\omega) = T\rho_0\left[1 + \frac{iR_A\Omega}{\omega\rho_0 T}G(R,r,L,T)\right], \tag{9}$$

where $G(R,r,L,T) = \sqrt{\left(1-\frac{4iT\eta\rho_0\omega}{R_A^2R^2(1-r)^2\Omega^2}\right)}$, $\Omega = \pi R^2(1-r^2)/L^2$, $R_A = \frac{8T\eta}{\Omega R^2 Y(r)}$,

$Y(r) = 1 + r^2-(1-r^2)/ln(1/r)$

and

Table 1
Complex density functions for four pore shapes.

Pore Shape	λ	$H(\lambda)$
slit (width $2b$)	$b\sqrt{(\omega/\nu)}$	$1-\tanh(\lambda\sqrt{-i})/(\lambda\sqrt{-i})$
cylinder (radius a)	$a\sqrt{(\omega/\nu)}$	$1-(2/\lambda\sqrt{i})J_1(\lambda\sqrt{i})/J_0(\lambda\sqrt{i})$
equilateral triangle (side d)	$(d\sqrt{3}/4)\sqrt{(\omega/\nu)}$	$1-3\coth(\lambda\sqrt{-i})/(\lambda\sqrt{-i}) + 3i/\lambda^2$
rectangle (sides $2a$,$2b$)	$\frac{2ab}{\pi\sqrt{(a^2+b^2)}}\sqrt{(\omega/\nu)}$	$\frac{-4i\omega}{\mu a^2b^2}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\{\alpha_k^2\beta_l^2[\alpha_k^2+\beta_l^2-(\frac{i\omega}{\mu})]^{-1}\}$, $\alpha_k = (k+\frac{1}{2})(\frac{\pi}{a})$, $\beta_l = (l+\frac{1}{2})(\frac{\pi}{b})$