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Technical note

Enhanced transmission loss in acoustic materials with micro-membranes

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A B S T R A C T

Due to their strong transmission loss (TL) at low frequencies, acoustic micro-membranes (AMMs) have been the subject of many studies. In this paper, a theoretical model for AMMs is established which gives a direct insight into their sound insulation qualities. Analysis of the model's acoustics shows how the sound insulation provided by a membrane can be significantly enhanced, and reveals the intrinsic characteristic of AMMs in mid-low frequency range. Further analysis on two AMMs separated by an air gap gives a quantitative description on how performance varies with the air gap. Using the theoretical model developed here, it is a straightforward matter to adjust an AMM's parameters to achieve a desired level of attenuation. This work opens the way for the potential application of AMMs in noise control engineering.

1. Introduction

Light-weight materials are extensively used in construction and industry [1]. However, they have one overwhelming disadvantage – low sound transmission loss – and this limits their range of applications. According to the classic theory of sound insulation, TL increases with mass per unit area [2,3], and this obstinate 'mass law' prevents the development of light-weight but high-TL materials.

The development of metamaterials has led acousticians to investigate the acoustic properties of small structures [4–8]. Remarkably, when AMMs are investigated [9–15], some of the results show a novel phenomenon: although of light weight, the membranes show high TL at low frequencies [16].

Lee and colleagues showed in 2009 that when a membrane was arranged in layers, high TL occurred below 735 Hz [17]. Recently, in 2015, Sui and colleagues described the acoustic properties of a single layer membrane applied on top of a honeycomb panel [16]. The TL of this configuration could reach 30 dB below 150 Hz while the mass per unit area of the whole composite was only 1.5 kg/m^2. This achievement has attracted considerable attention, and a number of studies have followed [18,19]. One recent study has shown that by adjusting various parameters the TL can be improved even further [18].

As have been demonstrated [16–18], the abnormal behaviors of AMMs can be regarded as the result arising from the membrane's apparent negative density at low frequencies. In this paper, this remarkable performance is theoretically explained in a contrary way. In Secs. II and III, an acoustical analysis of AMMs is presented and the fundamental reason for their anomalous TL is provided. Such theoretical analysis reveals the underlying link between the AMMs and traditional materials. Finally, discussions and conclusions are given in Sec. IV.

2. Sound insulation of one micro-membrane

Fig. 1(a) shows the geometry of the AMM investigated in this paper. It is made up of a regular array of circular holes in a rigid frame which is covered by a rubber membrane. The rigid frame clamps the flexible membrane in place at the circumference of the holes, which each have diameter $2r$. To investigate the metamaterial's sound transmission qualities, it is considered to be made up of identical unit cells of membrane-covered holes, as shown in Fig. 1(a). To derive the acoustic properties of each unit cell, two assumptions are made: (1) the dimensions of the membrane are far less than the wavelength of the incident sound, which is the intrinsic characteristics of a metamaterial that is constructed in a subwavelength scale; (2) the frame is regarded as a rigid boundary.

From these two assumptions, we can use an electro-mechanical analogy to describe the propagation of sound through one unit cell of the micro-membrane. Notice that, in this paper, the proposed structure is presented as a 2D arrangement of circular membranes clamped over a rigid frame, rendering a 3D acoustic environment of the whole sample. However, the configuration of each unit ensures that it generates a symmetric mode, which indicates that we can considerably reduce the complexity of our study by just

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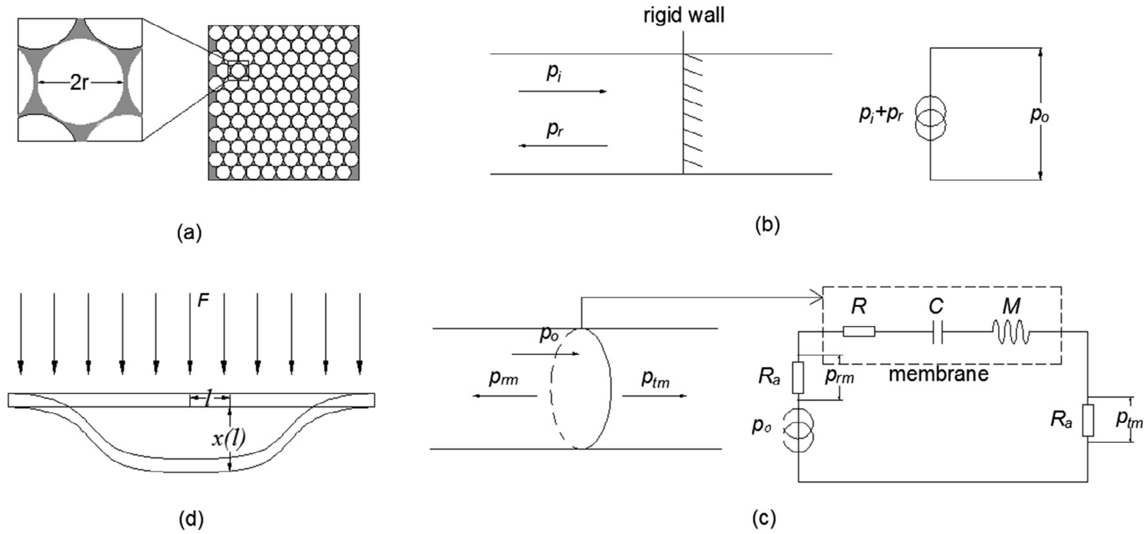

Fig. 1. (a) Metamaterial made of an array of circular holes of radius r and clamped inside a rigid frame. Each circle, of diameter $2r$, is a unit cell. (b) Sound propagation in a rigid-terminal tube. (c) Transmission of sound through one unit cell, where p_{tm} and p_{rm} represent sound pressure radiated caused by membrane vibration. And equivalent circuit based on electro-mechanical analogy; (d) Deformation $x(l)$ of a clamped membrane acted on by a uniform force F. The distance from the centre of the membrane is l.

considering a unit mounted in a 1D waveguide [20]. Fig. 1(b) shows the sound transmitting through a tube with a rigid–wall terminal. In equivalent circuit, it represents the open-circuit voltage p_0. Due to the rigid–wall terminal, the open-circuit voltage can be obtained as [2]

$$p_0 = p_i + p_r = 2p_i, \tag{1}$$

where p_i and p_r are the input and reflected sound pressure near the terminal, respectively.

Fig. 1(c) shows the path taken by a sound wave as it passes through the membrane. The membrane's effective mass, stiffness, and damping are represented as M_{mem}, K_{mem}, and R_{mem}. Herein, p_0 represents the sound pressure on the membrane (a sum of incident and reflected pressure), while p_{tm} and p_{rm} are radiated sound caused by membrane vibration, in which p_{tm} actually represents the transmitted sound pressure. R_a designates the characteristic impedance of air so that $R_a = \rho_0 c_0$, where ρ_0 and c_0 respectively represent air density and sound velocity in air. Define $M = M_{\mathrm{mem}}/S$, $C = S/K_{\mathrm{mem}}$, and $R = R_{\mathrm{mem}}/S$, representing the acoustic impedance of one unit cell, where S is the area of one unit cell so that $S = \pi r^2$. Then the transmitted sound pressure can be expressed as

$$p_{tm} = \frac{2p_i R_a}{R + 1/(jwC) + jwM + 2R_a}. \tag{2}$$

where w is the angular frequency of the sound wave. The sound pressure transmission coefficient can be expressed as $t_p = p_{tm}/p_i$. The sound power transmission coefficient t_i and sound transmission loss TL can be obtained as:

$$t_i = |t_p|^2 = \left|\frac{2R_a}{R + 1/(jwC) + jwM + 2R_a}\right|^2 = \frac{4R_a^2}{[wM - 1/(wC)]^2 + (R + 2R_a)^2}, \tag{3}$$

$$TL = 10\log\left(\frac{1}{t_i}\right) = 10\log\left(\frac{[wM - 1/(wC)]^2 + (R + 2R_a)^2}{4R_a^2}\right). \tag{4}$$

Eq. (4) shows the fundamental resonant frequency occurs when $[wM - 1/(wC)] = 0$; that is, when

$$w_0 = \sqrt{1/(MC)}, \tag{5}$$

the TL dip of the unit cell occurs.

The value C can be obtained from K_{mem}, which refers to the membrane effective stiffness; it can be calculated from force F and average displacement $\bar{x}$ as [21]:

$$K_{mem} = F/\bar{x}, \tag{6}$$

where F is the force acting on the membrane due to a sound wave traveling through the membrane and generating a sound pressure difference across it. The deformation x resulting from F is illustrated in Fig. 1(d). Using thin-plate deformation theory, the membrane displacement corresponding to distance l from the centre $x(l)$ can be expressed as

$$q = H \cdot \nabla^4 x(l), \tag{7}$$

where q is the force per unit area on the membrane so that $q = F/S$. H is the flexural rigidity of the thin plate, so that $H = Et^3/[12(1-\mu^2)]$, where E is Young's modulus, μ is Poisson's ratio, t is thickness. In addition, the displacement $x(l)$ can also be written as [22]

$$x(l) = \frac{r^4\left(l^2/r^2 - 1\right)^2}{64H} q. \tag{8}$$

and the average displacement is

$$\bar{x} = \frac{1}{\pi r^2}\int_0^r x(l) \cdot 2\pi l \mathrm{d}l = \frac{r^4 q}{192H}. \tag{9}$$

Combining Eqs. (6) and (9), the stiffness of membrane can be obtained:

$$K_{mem} = \frac{16\pi E t^3}{r^2(1-\mu^2)}. \tag{10}$$

Actually the effective mass can be derived from the averaged kinetic energy. Notice that the vibration displacement η can be written in a form similar to Eq. (8), as

$$\eta = q\frac{r^4(l^2/r^2 - 1)^2}{64H} e^{jw\tau}. \tag{11}$$

Now consider the membrane to be made up of a large number of rings of width dl, then the kinetic energy of each ring can be calculated by