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Infinite-dimensional SVD for revealing microphone array's characteristics

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ABSTRACT

Nowadays, many acoustical applications utilize microphone arrays whose configurations have a lot of varieties including linear, planar, spherical and random arrays. Arguably, some configurations are better than the others in terms of acquiring the spatial information of a sound field (for example, a spherical array can distinguish any direction of arrival, while a linear array cannot distinguish the direction perpendicular to its aperture direction due to the rotational symmetry). However, it is not easy to compare arrays of different configurations because each array has been treated by a specific theory depending on the configuration of the array. Although several criteria have been proposed for evaluating and/or designing the arrays, most of them are application-oriented criteria, and the best configuration for some criterion may not be a better one for the other criterion. Therefore, an analysis method for microphone arrays which does not depend on the array configuration or application is necessary. In this paper, the infinite-dimensional SVD is proposed for analyzing and comparing the properties of arrays. The singular values, functions and vectors obtained by the proposed method provide the fundamental properties of an array.

1. Introduction

A microphone array is a quite fundamental device for acquiring spatial information of a sound field. A lot of signal processing methods, which effectively utilize the spatial information of sound, have been investigated for many applications including the direction-of-arrival estimation, sound enhancement, and source separation.

Nowadays, various configuration of microphone arrays are used in the above applications, such as linear, planar, spiral, spherical, and random configurations. Arguably, some array configurations are better than the others in terms of acquiring the spatial information of a sound field. However, it is not easy to compare the arrays of different configurations because each array has been treated by a specific theory depending on the configuration of the array. For example, spherical arrays are usually treated by the spherical coordinate, while the other configurations are treated by the Cartesian coordinate. As another example, although the spherical harmonics is often utilized as an useful tool for analyzing spherical arrays, it may not be effective for the other configurations. Therefore, an analysis method for microphone arrays which does not depend on the array configuration is necessary for the comparison.

Several criteria have been proposed for microphone arrays in the context of evaluating and/or designing the arrays [1–16]. By quantifying the effect of the array signal processing methods including beamforming [1–3,7–15], time-difference-of-arrival (TDOA) estimation [4], near-field acoustical holography (NAH) [6] and sound field interpolation [16], those criteria enable optimization of microphone positions to achieve better performance [1–4, 6–8,10,11,13–16]. Although they have been proved to be useful in the literatures, there is one limitation: they are application-oriented criteria. That is, the best configuration for some criterion specialized to a single processing method, say beamforming, may not be a better configuration for the other applications such as NAH. On the one hand, application dependent assessment is useful for optimizing the performance of a specific processing method. On the other hand, an application *independent* criterion of characteristics of microphone arrays should also be important for comparing and analyzing the properties of the arrays. While a few research considers analysis of array configurations without restriction to a specific application [5], restriction on the class of configurations (such as linear, planar, spherical or random) may exist. To the best of our knowledge, there have been no analysis method depending on neither application nor class of configurations.

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In this paper, a continuous analogous of the singular value decomposition (SVD), namely *infinite-dimensional SVD*, is proposed for analyzing the microphone arrays without any restriction on the application or class of configurations.[1] SVD is a factorization of a matrix for revealing the properties of the linear transformation associated with the matrix. Since signals of any sound field acquired by a microphone array can be represented by a certain linear transformation [see Eq. (19)], SVD of that transformation obtains information of the sampling property of the microphone array which does not depend on the application or configuration. Our proposal is to provide a method of calculating this SVD which can be a fundamental tool for array analysis. This linear transformation is a mapping from an infinite-dimensional space[2] to a finite dimensional space, and thus we refer to the proposed SVD of an infinite-dimensional matrix associated with this transformation as the infinite-dimensional SVD. In addition to the infinite-dimensional SVD, for visualizing the property of an array, *spatial impulse response* is also proposed as an intermediate representation of the information acquired by a microphone array.

2. Sampling of sound field and its plane wave representation

A microphone array is the device which measures sound pressures at several points where the microphones are placed. This can be modeled by the sampling of a sound field which obeys the corresponding physical model. In linear acoustics, sound propagation is modeled by the wave equation,

$$\left(\triangle - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)p(\boldsymbol{x},t) = 0, \tag{1}$$

where $\triangle = \sum_n \partial^2/\partial x_n^2$ is the Laplace operator, t is time, $\boldsymbol{x} = (x_1, x_2, x_3)$ is position, p is sound pressure, and c is the speed of sound. The Fourier transform on time variable $\mathcal{F}_t$ converts Eq. (1) into the Helmholtz equation:

$$\left(\triangle + k^2\right)u(\boldsymbol{x},\omega) = 0, \tag{2}$$

where $u = \mathcal{F}_t p, k = \omega/c$ is the wave number, and ω is the angular frequency. For simplicity, the dependency on ω will be omitted hereafter as $u(\boldsymbol{x})$. Although the discussion in this paper will rest on the Helmholtz equation, this does not mean that the proposed analysis will be restricted to the narrowband applications only, because a solution to the Helmholtz equation (in the frequency domain) can be converted back to the corresponding solution of the wave equation (in the time domain) via the inverse Fourier transform.

Observation of a sound field by M microphones is a mapping from the sound to measured signal, which is defined as the sampling operator S_M as

$$S_M : u(\boldsymbol{x}) \mapsto \{u(\boldsymbol{x}_m)\}_{m=1}^M, \tag{3}$$

where $\{u(\boldsymbol{x}_m)\}_{m=1}^M$ is the sound field u sampled by M microphones placed at $\{\boldsymbol{x}_m\}_{m=1}^M$. The sampling property of the microphone array depends on the positions of the M microphones $\{\boldsymbol{x}_m\}_{m=1}^M$. Note that this sampling operator maps a function, which lies in an infinite-dimensional space, to an M-dimensional vector. That is, only M-dimensional components of the sound field u (over the infinite-dimensional components) are retained in the observed signal as illustrated in Fig. 1. We will describe this M-dimensional subspace of the infinite-dimensional space as *measurable information*. Since information not contained in the observed signal cannot be utilized in any application, knowing which M-dimensional components are measurable (are contained in the measured signal) is important.

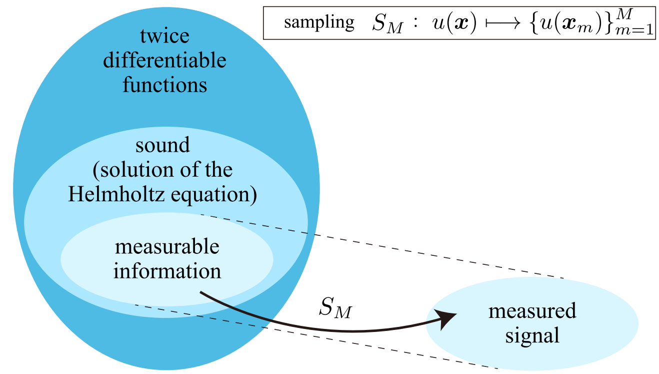

Fig. 1. Schematic of the sound field sampling procedure. S_M is the sampling operator to be analyzed. The aim of the proposed method is to reveal the measurable information, which is the M-dimensional components of the sound field retained in the measured signal, of a given M-channel microphone array.

The aim of this paper is to analyze the sampling property of a microphone array based on the above physical model. That is, the properties of the sampling operator S_M are analyzed in order to reveal the M-dimensional components (measurable information) of the sound field contained in the observation. However, S_M cannot be analyzed directly because the sound field lies in the infinite-dimensional space which is intractable by the finite-number of calculations. Fortunately, the sound field can be represented by a tractable model using plane waves thanks to the restriction of the class of functions due to the Helmholtz equation as shown in Fig. 1. That is, we do not have to consider all twice-differentiable functions which is difficult to handle, but only part of them is sufficient that enables a simple characterization of all sound fields.[3]

2.1. Plane wave approximation of sound field

It is well known that any solution to the homogeneous Helmholtz equation can be approximated arbitrarily well by the linear combination of plane waves [19]:

$$u(\boldsymbol{x}) \approx \tilde{u}(\boldsymbol{x}) = \sum_{n=1}^{N} \alpha_n \exp(jk\langle \boldsymbol{x}, \boldsymbol{v}_n\rangle), \tag{4}$$

where $j = \sqrt{-1}, \langle\cdot,\cdot\rangle$ is the standard inner product, $\alpha_n \in \mathbb{C}$ is the expansion coefficient which depend on $\tilde{u}$ and $\{\boldsymbol{v}_n\}_{n=1}^N$, and $\{\boldsymbol{v}_n\}_{n=1}^N \subset \mathbb{S}^2$ is a set of the unit vectors whose directions are uniformly distributed. By defining a linear operator

$$H : \{\alpha_n\}_{n=1}^N \mapsto \tilde{u}(\boldsymbol{x}), \tag{5}$$

Eq. (4) can be written as

$$\tilde{u}(\boldsymbol{x}) = H\boldsymbol{\alpha}, \tag{6}$$

where $\boldsymbol{\alpha}$ is an N-dimensional column vector defined as

[1] The preliminary version of this work, without the contents in Sections 4 and 6 of this paper, has appeared in the conference proceeding [17].

[2] The dimension of a functional space, which is the space of the square-integrable functions on the unit sphere in this paper, is usually infinite because the number of the basis elements is not finite.

[3] Note that this is not a rigorous mathematical statement, for example, the space of the twice differentiable functions is too restricted as it is not complete. The derivatives should be understood in a suitable weak sense as in [18]. However, such details are not important in this paper because the solution space of the Helmholtz equation will not be treated directly as later shown in Section 3. Therefore, the mathematical details are omitted here in order to keep the discussion accessible for wider audiences.