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Research paper

Universal solution for the characteristic time and the degree of settlement in nonlinear soil consolidation scenarios. A deduction based on nondimensionalization

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ABSTRACT

Using the mathematical-physical process of nondimensionalization of governing equations, the problem of nonlinear soil consolidation based on the Cornetti and Battaglio model removing some of the restrictive hypotheses assumed by these authors is studied for the search of the dimensionless groups that govern the characteristic time and the average degree of settlement. The derived groups, once verified by numerical simulations, allow the universal representation of the aforementioned unknowns for the wide range of properties of real soils.

1. Introduction

Theory of primary soil consolidation is focused to the determination of the degree of settlement of structures resting on saturated soil (clay) layers, as well as the time required to reach the final steady settlement [7,26]. In the consolidation process, the initial excess pore pressure set up in the water by the rapidly applied load dissipates over time, as water is drained from the soil, causing some settlement due to changes in volume and transferring part of the load to the soil skeleton. In particular, when the loading area is larger in comparison to the thickness of the clay, the settlement can be considered as a one-dimensional phenomenon, hypothesis assumed in this paper. In this work we research the nonlinear original consolidation model of Cornetti and Battaglio [14] (C–B hereinafter), as well as other less restrictive models based on the same C–B constitutive dependences (deleting some of its more severe hypotheses), with the aim of determining by the technique of nondimensionalization: i) the dimensionless groups that rule the solution of this model and, ii) from these groups and for a wide range of properties of real soils, the universal curves for the characteristic time and the average degree of settlement (percentage of settlement produced on the soil surface). C–B and later Arnod et al. [4], using the same model, following a less formal procedure, only report the average degree of pressure dissipation (a variable of relative interest) and say nothing about the characteristic time.

The characteristic time, a reference to make time a dimensionless variable, needs to be defined both in terms of pressure dissipation and settlement since these variables may appreciably differ due to the nonlinearity of the problem. To assign an unmistakable physical meaning, characteristic time is taken as the time required by the process for an appreciable fall (90%)

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of the excess pore pressure (averaged over the entire domain), or for the development of the final settlement. In this way, the characteristic time and the duration of the consolidation process are closely related to each other.

Setting aside other variants of the nonlinear consolidation problem [9,17,27] it is worthy to mention the Juárez-Badillo model [19], which uses constitutive relations different from those of C–B for the changes in void ratio and permeability with effective pressure. Both models (C–B and Juárez-Badillo), which assume the severe hypotheses of constant soil thickness in the term of contraction of the governing equation (term that describes the temporal change of the relative void ratio) and constant thickness for the volume element (hypotheses related one to another), are solved numerically by the authors who provide very conservative solutions – fortunately always on the security side – that converge to the same results in problems with small changes in these parameters (quasilinear problems).

Although the nondimensionalization or scale analysis techniques are current and successful tools applied by many researchers in the search for dimensionless groups that govern a large number of linear problems of science and engineering, mainly for the development of scale prototypes or 'scaling' (especially in fields such as fluid dynamics and heat transfer [5,6,10]), to our knowledge very few applications of this technique to mathematical models defined by nonlinear equations have been found in the literature until now [13]. The main reason, undoubtedly, is the inherent lack of homogeneity in the mathematical model – for example, the existence of logarithmic or other types of functions contained in the governing equations. However, this should not be a formal argument since this kind of functions would be linearized, assigning to the emergent coefficients units of measurement. Rather, the reason would lie in the complicated steps – as in this and other works [20,23] – that lead to the derivation of the precise dimensionless groups from the dimensional governing equations. Among these steps are: i) the appropriate choice of references (some of them do not set in the problem statement), and ii) the averaging of the resulting dimensionless governing equations.

Through the nondimensionalization process, the large number of dimensionless isolated parameters, together with the rest of the geometric and physical characteristics and the hidden references, is reduced to the minimum set of independent groups that best help the researchers to manipulate the solution. The remainder is made by the pi theorem [8]: all the unknowns of interest, expressed in their dimensionless form, depend on these groups and eventually on the dimensionless independent variables. As it is known, this theorem, derived from the theory of homogeneous functions, is based on the invariance of the solution with regard to a change in the measurement units.

It is worth mentioning here the essential difference between the dimensional analysis and the nondimensionalization of the governing equations, two techniques whose purpose is the same, namely, the derivation of dimensionless groups. The first one [16,11] starts from the list of relevant variables that influences the phenomenon. Once these are expressed as dimensional equations in terms of the primary quantities (length, mass and time), the dimensionless groups affecting the solution are derived through simple mathematical manipulation. Thus, dimensional analysis gives the most general form of the solution whatever is the model used. On the other hand, nondimensionalization [3,10] starts from the governing equations (including the boundary conditions) and, by introducing suitable references, the dependent and independent variables are changed to their dimensionless form; from the resulting normalized equation, the dimensionless groups are derived after more or less complicated mathematical steps. The solution obtained by the nondimensionalization procedure, being related to the specific model used, falls within the general expression provided by dimensional analysis. Undoubtedly, the last technique leads to the most accurate solution as demonstrated in many works [21,28].

To verify the results of this research, numerical simulations are performed based on the network method [18]. As expected, for the same value of the groups, the characteristic time does not change and the average degree of settlement (or pressure dissipation) related to a same dimensionless time also retains the same value.

2. Nomenclature

c_k	permeability index (dimensionless)
c_v	consolidation coefficient (m^2/s or m^2/year)
$c_{v,o}$	initial consolidation coefficient (m^2/s or m^2/year)
dz	element of differential length in the direction of the spatial coordinate z (m)
dz_o	element of differential length in the direction of the spatial coordinate z at the initial time (m)
I_c	compression index (dimensionless)
e	void ratio (dimensionless)
e_o	initial void ratio (dimensionless)
e_f	final void ratio (dimensionless)
e_m	mean value of the void ratio (dimensionless)
H	soil thickness up to the impervious boundary or drainage length (m)
H_o	initial soil thickness (m)
H_f	final soil thickness (m)
k	hydraulic conductivity (or permeability) (m/s or m/year)
k_o	initial hydraulic conductivity (m/s or m/year)
k_f	final hydraulic conductivity (m/s or m/year)
L(10)	natural logarithm of 10 (dimensionless)
m_v	coefficient of volumetric compressibility (m^2/N)