Commun Nonlinear Sci Numer Simulat 54 (2018) 1–12

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Research paper

Research on the reliability of friction system under combined additive and multiplicative random excitations

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ARTICLE INFO

Article history:
Received 7 November 2016
Revised 20 April 2017
Accepted 18 May 2017
Available online 19 May 2017

Key words:
Friction system
Stochastic excitations
Stochastic averaging method
First passage failure
Reliability

ABSTRACT

In this paper, the reliability of a non-linearly damped friction oscillator under combined additive and multiplicative Gaussian white noise excitations is investigated. The stochastic averaging method, which is usually applied to the research of smooth system, has been extended to the study of the reliability of non-smooth friction system. The results indicate that the reliability of friction system can be improved by Coulomb friction and reduced by random excitations. In particular, the effect of the external random excitation on the reliability is larger than the effect of the parametric random excitation. The validity of the analytical results is verified by the numerical results.

1. Introduction

Friction, which is one of the typical non-smooth factors, exists widely in people's life and engineering practice, such as robot [1,2], clutch, computerized numeric control (CNC) [3], automobile tires, brake pads, dry friction damper [4]. It may dramatically change the dynamic mechanical behaviors, and even induce the structural insecurity. Grasping the dynamic characteristics of friction can help to identify the causes of unwanted behavior such as squeal of car brakes and to reduce the harm induced by the friction force. Therefore, for many years the topic of friction has been received widespread attention [5–9].

Random factors exist widely in the physical system and may markedly affect the dynamic behavior of the physical system. Whereas, there are few researches on the friction system under random excitations, and current works almost focus on the numerical solutions. Sun [10] researched the random response of Coulomb friction system by applying the generalized cell mapping method which is based on the short-time Gaussian approximation. Feng [11] used a two-dimensional mean Poincare map to establish the discrete model of random friction system and studied the stochastic stick-slip. Brouwers [12] investigated the non-linearly damped response of a marine riser subject to random waves. The equivalent nonlinear method was utilized by Tian [13] to study the optimal load resistance of a randomly excited nonlinear electromagnetic energy harvester with Coulomb friction.

Reliability is important in many practical applications, such as the comfort of the vehicle bump vibration, the security of the building under the wind load or the impact of the earthquake, and others. It is one of the most essential and difficult problems in random vibration theory. In the structural systems, the reliability problem is usually the first passage problem. To study the reliability, one should solve the diffusion processes related to the response of a stochastic system. However, it is hard to find out the analytical results of diffusion processes, even for the case of one dimension, the known analytical

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http://dx.doi.org/10.1016/j.cnsns.2017.05.014
1007-5704/

results are limited. Therefore, some scholars have developed many numerical methods, such as finite difference method, finite element method and generalized cell mapping method [14–16]. Based on the stochastic averaging method, the dimension of a stochastic system can be reduced, which makes further research on the reliability be possible. And many scholars have studied the reliability of a system under random excitations by applying the stochastic averaging method [17–22]. Chen [23] used the stochastic averaging method to investigate the first passage failure of quasi integrable-Hamiltonian systems under combined harmonic and white noise excitations. This method was utilized by Li [24] to study the first passage problem for strong nonlinear stochastic dynamical system. Non-smooth factors are ubiquitous in modern engineering structures. And their existence may make the dynamics behavior of a system more complex. Some scholars tried to apply the stochastic averaging method to study the stochastic responses of non-smooth system. Feng [25] and Zhao [26] researched the stochastic responses of vibro-impact system under additive and multiplicative random excitations. Wu [27] investigated the stationary response of multi-degree-of-freedom vibro-impact systems to Poisson white noises. However, the reliability of non-smooth system is rarely investigated. Inspired by previous work, we tried to use the method to investigate the reliability of friction non-smooth system.

The reliability of a system can be measured by many indicators. In this paper, the reliability function, the conditional probability density function (PDF) of first passage time and the mean first passage time are chosen. The averaged Itô stochastic differential equation for the energy of friction system is first derived by using the stochastic averaging method. Then, the Backward Kolmogorov (BK) equation for the conditional reliability function and the Generalized Pontryagin (GP) equations for the moments of first passage time are established. Finally, the analytical results, which are obtained by solving the BK equation and GP equations, are comparing with the numerical results to verity the validity of the analytical method.

2. Description of stochastic friction system and average equation

Consider the non-linearly damped Coulomb friction system under combined additive and multiplicative Gaussian white noise excitations,

$$\ddot{x}+{\omega_0}^2x+2c_1\dot{x}+c_2\dot{x}^3+f_k\mathrm{sgn}(\dot{x})=\xi_1(t)+x\xi_2(t) \tag{1}$$

where x, $\dot{x}$, $\ddot{x}$ are displacement, velocity and acceleration, respectively. The dot $\cdot$ represents the differentiation with respect to the time t. ω_0 is the nature frequency of the system, c_1, c_2 are small parameters. f_k is the amplitude of friction and $\mathrm{sgn}(\boldsymbol{.})$ represents the signum function.

$\xi_1(t)$, $\xi_2(t)$ are independent Gaussian white noise which satisfy the following conditions

$$E(\xi_k(t))=0,\ k=1,2,$$

$$E(\xi_k(t)\xi_l(t+\tau))=2D_{kl}\delta(\tau),\ k,l=1,2.$$

Introducing the transform $x_1=x$, $x_2=\dot{x}$ and considering Wong–Zakai approximation, Eq. (1) is equivalent to the following Itô stochastic differential equations

$$\begin{aligned}dx_1&=x_2dt,\\ dx_2&=m(x_1,x_2)dt+\sigma(x_1,x_2)dW(t),\end{aligned} \tag{2}$$

where $W(t)$ is standard Wiener stochastic process,

$$m(x_1,x_2)=-{\omega_0}^2x_1-2c_1x_2-c_2{x_2}^3-f_k\mathrm{sgn}(x_2),$$

$$\sigma^2(x_1,x_2)=2D_{11}+2D_{22}{x_1}^2.$$

The integral equation of motion of undamped system is as the following equation

$$H=\frac{1}{2}{\omega_0}^2{x_1}^2+\frac{1}{2}{x_2}^2. \tag{3}$$

According to Eqs. (2) and (3), the Itô stochastic differential equation governing energy can be obtained by using the Itô formula

$$\begin{aligned}dH=&\left[\mp\sqrt{2H-2G(x_1)}f\left(\pm\sqrt{2H-2G(x_1)}\right)+\frac{1}{2}\sigma^2\left(x_1,\pm\sqrt{2H-2G(x_1)}\right)\right]dt\\ &\pm\sqrt{2H-2G(x_1)}\sigma\left(x_1,\pm\sqrt{2H-2G(x_1)}\right)dW(t),\end{aligned} \tag{4}$$

where $G(x_1)$ is the potential energy,

$$G(x_1)=\frac{1}{2}{\omega_0}^2{x_1}^2,$$

$$f(x)=-2c_1x-c_2x^3-f_k\mathrm{sgn}(x).$$

Using the stochastic averaging method [28,29], one can obtain the averaged Itô stochastic differential equation for total energy of system (1)

$$dH=\bar{m}(H)dt+\bar{\sigma}(H)dW(t), \tag{5}$$