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Research paper

Stability analysis of neural networks with time-varying delay: Enhanced stability criteria and conservatism comparisons

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ABSTRACT

This paper is concerned with the stability analysis of neural networks with a time-varying delay. To assess system stability accurately, the conservatism reduction of stability criteria has attracted many efforts, among which estimating integral terms as exact as possible is a key issue. At first, this paper develops a new relaxed integral inequality to reduce the estimation gap of popular Wirtinger-based inequality (WBI). Then, for showing the advantages of the proposed inequality over several existing inequalities that also improve the WBI, four stability criteria are derived through different inequalities and the same Lyapunov–Krasovskii functional (LKF), and the conservatism comparison of them is analyzed theoretically. Moreover, an improved criterion is established by combining the proposed inequality and an augmented LKF with delay-product-type terms. Finally, several numerical examples are used to demonstrate the advantages of the proposed method.

1. Introduction

In the past few decades, neural networks have attracted much attention because of their wide spread applications in pattern recognition [1], associative memories [2] and other scientific areas [3]. These applications are heavily dependent on the stability of the designed neural networks, while, time delays, inevitably existing in many neural networks, are usually a cause of oscillation and instability. Thus, the stability analysis of the delayed neural networks (DNNs) has become a hot topic in both theory and practice, and various approaches have been proposed for this issue [4–47].

It is well known that the criteria with less conservatism can provide larger admissible delay regions guaranteeing the stability of DNNs (i.e, the provided values are more closed to the analytical values). Thus, much weight in the field of stability analysis of DNNs has been put on the reduction of conservatism of the stability criteria. So far, most results are developed via the Lyapunov stability theory [4–36]. During the development of stability criterion through this theory, constructing a suitable Lyapunov–Krasovskii Function (LKF) and effectively estimating its derivative are two key points related to the conservatism of stability criteria. From the procedure of stability criteria development, our previous work [27] has summarized most the related techniques and respectively analyzed their contributions to reduce the conservatism. It is found that the estimation of the integral terms in the derivative of LKF is one of the most important step.

In the stability analysis of DNNs, many approaches have been developed for estimating the integral terms, and some of them are reviewed in the follows. Model transformations with the cross-term bound inequalities were used to han-

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Table 1
Notations and abbreviations used in this paper.

$\mathcal{R}^{n\times m}$	The set of all $n \times m$ real matrices
N^T (N^{-1})	The transpose (inverse) of the matrix N
$P > 0$ (≥ 0)	P is a real symmetric and positive-definite (semi-positive-definite) matrix
diag$\{\cdots\}$	A block-diagonal matrix
I (0)	The identity (zero) matrix
Sym$\{X\}$	$X + X^T$
$\begin{bmatrix} X & Y \\ * & Z \end{bmatrix}$	$\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix}$
DNNs	DNNs
DNNs	Delayed neural networks
LKF	Lyapunov–Krasovskii function
LMI	Linear matrix inequality
WBI	Wirtinger-based inequality
AFBI	Auxiliary function-based integral inequality
FMBI	Free-matrix-based integral inequality
ERI	Extended relaxed integral inequality
RCCL	Reciprocally convex combination lemma
NDV	The number of decision variables
AMUB	Admissible maximal upper bound

dle the integral terms for deriving stability criteria of Cellular DNNs [4] and Hopfield DNNs [5]. After the development of free-weighting matrix approach [6], which overcomes the drawback of model transformations, the asymptotical and/or exponential stability analysis of DNNs were studied by using the free-weighting matrix approach and its improved form to estimate the integral terms [7–10]. In order to reduce the decision variables of free-weighting matrix based criteria, the Jensen inequality was increasedly used for estimating the integral terms [11–21], especially after the development of reciprocally convex combination lemma (RCCL). Later, the Wirtinger-based inequality (WBI) became the most popular method for the analysis of various DNNs [22–30], as the WBI is tighter than the Jensen inequality and improves the Jensen inequality for the task of integral term estimation.

Recent years, many new integral inequalities are developed to improve the WBI from different directions. Firstly, the free-matrix-based integral inequality (FMBI), which encompasses the WBI as a special case [45], was extended to the DNNs for deriving the improved criteria [31–33,51]. Meanwhile, the auxiliary function-based integral inequality (AFBI) [44] reduces the estimation gap of the WBI by introducing additional terms. However, both the FMBI and the AFBI require the increase of decision variables for reducing the conservatism. Very recently, a relaxed integral inequality proposed in [46] provides a new way to improve the WBI without requiring any extra slack matrix. While this inequality still has some drawbacks (see Remark 7) and can be further improved. On the other hand, as mentioned in [52], it is not enough to judge the conservatism of the obtained criteria just through the inequalities used since the better inequality does not always lead to the less conservative criterion. On the contrary, the conservatism analysis from the criteria themselves, instead of inequalities used, should be carried out for showing the advantages of the developed criteria of the DNNs. While few literature has done similar work. Those two considerations motivate the present research.

This paper further investigates stability analysis for DNNs with an interval time-varying delay, and the objective is to develop a new way to enhance the WBI-based stability criteria. Firstly, an extended relaxed integral inequality (ERI) is developed to estimate the integral terms in the derivative of LKF. It not only reduces the estimation gap of the WBI but also improves the inequality proposed in [46] by introducing new terms (See Remarks 1 and 3). Then, in order to clearly check the contribution of the proposed ERI in comparison with the typical inequalities, different inequalities are applied to develop several stability criteria (Theorems 1–4) under the same LKF, and also the conservatism analysis for those criteria is carried out theoretically. Moreover, the proposed inequality is also used to develop a less conservative criterion (Theorem 5) by constructing a new LKF with several delay-product-terms and a relaxed self-positive condition. Finally, several numerical examples are given to illustrate the advantages of the proposed method.

The reminder of the paper is organized as follows. Section 2 gives problem formulation and presents the ERI. Several stability criteria are developed in Section 3. In Section 4, several numerical examples are used to demonstrate the effectiveness and superiority of the proposed criteria. Conclusions are given in Section 5.

Table 1 shows the list of notations and abbreviations used throughout this paper.

2. Problem formulation and preliminary

Consider the following neural network with an interval time-varying delay:

$$\dot{z}(t) = -Az(t) + W_0 f(W_2 z(t) + J) + W_1 f(W_2 z(t - d(t)) + J) \tag{1}$$

where $z(t) = [z_1(t)\ z_2(t)\ \ldots\ z_n(t)]^T$ is the neuron state vector; $f(\cdot) = [f_1(\cdot)\ f_2(\cdot)\ \ldots\ f_n(\cdot)]^T$ represents the neuron activation function; $A = \text{diag}\{a_1, a_2, \ldots, a_n\} > 0$ is the feedback gain; W_0, W_1 and $W_2 = [W_{21}^T, W_{22}^T, \ldots, W_{2n}^T]^T$ are the known interconnection weight matrices; $J = [J_1\ J_2\ \ldots\ J_n]^T$ is the constant vector; and the time delay, $d(t)$, is a continuous differentiable