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Research paper

Control effect of periodic variation on the growth of harmful algal bloom causative species

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ABSTRACT

Blue-green algae and *Dinoflagellate* etc. are common types of phytoplankton as causative species which cause the harmful algal blooms (HABs). The growth process of causative species is complex according to the variation of the environmental disturbance such as the periodic factor in reality and recent studies have not revealed the secret of the growth complexity yet. Based on the empirical and theoretical results of the growth of causative species, a nonlinear controlled system with periodic factor was obtained and the different effects of the periodic factor on the control of the cell density and the growth rate of causative species were studied by three theorems using the norm theory and finite difference method. Simulations and experimental data were also used to assess the effectiveness of the controlled results.

1. Introduction

Harmful algal blooms (HABs) are mainly produced by rapid growth of algal species named as the causative species with certain conditions and have severe impacts on public health and coastal ecosystems [1,2]. One of the most famous reproduction mode of the causative species for the unicellular algae [3,4] is the well-known fact that one cell divided into two cells by asexual reproduction. In addition, the rate of the growth is complicatedly and nonlinearly affected by different environmental factors such as the ratios of nitrogen to phosphorus [5], fluorescent and red light environments [6]. So the inherent regularity between the growth process of the causative species and the complex environmental factors still remains a mystery.

Nowadays, many works focus on the control of growth process for these causative species in HABs experimentally. For instance, the higher concentration of kanamycin was found to inhibit the growth velocity of unicellular green algae *Chlorella vulgaris* [7]. The effect of assemblage age on the uptake rate of algal species was studied for the incorporation of cells and

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eliminating differences in substrate roughness [8]. When three proteins named GsSPT1 (G. sulphuraria sugar and polyol transproter 1), GsSPT2 and GsSPT4 were studied, results were shown that the deletion of N-terminus of the protein in SPT1 affected the maximal transport velocity and released the dependency of the pH on sugar uptake, while the uptake by other two proteins was not active for the pH-dependence [9].

Moreover, according to the fact that the growth process of the causative species in reality becomes much more complex than that of growth under laboratory, the growth process can be described mathematically by dynamical systems and many interesting and important works have been obtained. For example, the dynamical behavior of algal growth with impulsive disturbance [10], such as chaos [11] and stability of attractors [12,13], was studied in colony formation, in the interaction growth and in the allelopathic of phytoplankton. In the light of the experimental results of the growth interaction such as toxin or non-toxic algae, organic pollutants, nutrients and bacteria, the coupled dynamical models were used to simulate the interactions [14–18]. The effect of allelopathic competition between two algal species on the growth rate of algae was also reported by coupled nonlinear models [19–21].

However, there exist few results about the effect of the environmental factor on the growth process of the causative species from the point view of control theory in term of the experimental results, data collection in field and the current mathematical models of causative species. In this paper, we concentrate on the control effect of the periodic environmental factor on the cell density and the growth rate of the causative species, in which the periodic environmental factor include variation of seasons, light intensity, temperature and some inner cycle clock, etc. [22,23]. Our work can play vital roles for revealing the secret of the HABs.

The paper is organized as follows. In Section 2, the nonlinear controlled system for the causative species growth is established and the stable condition of the system is presented. Three theorems about the relationships between the periodic factor and the cell density and the growth rate of the causative species are studied in Section 3, respectively. Simulations and experimental data in Section 4 and in Section 5 also illustrate the control effect of the periodic factor on the cell density and the growth rate of causative species theoretically and experimentally. Conclusions and discussions are given in Section 6.

2. Problem assumption

This section revolves around the conditions like exist and unique solution and stable theory for the nonlinear reaction-diffusion system with initial and boundary conditions based on the experimental results of growth of the causative species.

In general, the growth process of the causative species dynamically satisfies the following system:

$$\frac{\partial P(x,y,z,t)}{\partial t} = f(P(x,y,z,t),\alpha(x,y,z,t),\beta(x,y,z,t),u(x,y,z,t)), \tag{2.1}$$

where $P(x, y, z, t)$ is the cell density of the causative species,

$$P \in \ Hilbert\ space\ H([\xi,\eta],\mathbb{R}^n),$$

$$(x,y,z,t) \in \mathbb{R}^3 \times (0,+\infty),$$

$$f(P(x,y,z,t),\alpha(x,y,z,t),\beta(x,y,z,t),u(x,y,z,t))$$

is the factor including the birth rate $\alpha(x, y, z, t)$, the death rate $\beta(x, y, z, t)$ and the controller as $u(x, y, z, t)$, like gene modulating factors and the strength of sunlight, etc.

Based on the models in Ref.[24,25] and the fact that the main reproductive mode of single cell species in the causative species is binary fission and the growth velocity varies with different spatial location and the different environmental factors, the dynamical system (2.1) for the growth process of the causative species can be represented as follows:

$$\begin{aligned}\frac{\partial P(x,y,z,t)}{\partial t} &= f(P(x,y,z,t),\alpha(x,y,z,t),\beta(x,y,z,t),u(x,y,z,t))\\ &= (\alpha+u(t)-\beta(t))P(x,y,z,t)-(\alpha+u(t))\frac{P^2(x,y,z,t)}{P_A}+D\triangle P(x,y,z,t),\end{aligned} \tag{2.2}$$

where

$$(\alpha+u(t)-\beta(t))P(x,y,z,t)-(\alpha+u(t))\frac{P^2(x,y,z,t)}{P_A}$$

means the binary fission term,

$$D\triangle P(x,y,z,t) = D\left(\frac{\partial^2 P(x,y,z,t)}{\partial x^2}+\frac{\partial^2 P(x,y,z,t)}{\partial y^2}+\frac{\partial^2 P(x,y,z,t)}{\partial z^2}\right)$$

stands for the spatial diffusion term. Since the velocity of the cell division of the causative species is affected by different periodic factors like strength of sunlight, temperature and inner cycle clock, etc., we introduce the controller as