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Research paper

Grey-Markov prediction model based on background value optimization and central-point triangular whitenization weight function

Jing Ye[a,*], Yaoguo Dang[a], Bingjun Li[b]

[a] College of Economics and Management, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, PR China
[b] College of Information and Management Science, Henan Agricultural University, Zhengzhou 450000, PR China

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ABSTRACT

Grey-Markov forecasting model is a combination of grey prediction model and Markov chain which show obvious optimization effects for data sequences with characteristics of non-stationary and volatility. However, the state division process in traditional Grey-Markov forecasting model is mostly based on subjective real numbers that immediately affects the accuracy of forecasting values. To seek the solution, this paper introduces the central-point triangular whitenization weight function in state division to calculate possibilities of research values in each state which reflect preference degrees in different states in an objective way. On the other hand, background value optimization is applied in the traditional grey model to generate better fitting data. By this means, the improved Grey-Markov forecasting model is built. Finally, taking the grain production in Henan Province as an example, it verifies this model's validity by comparing with GM(1,1) based on background value optimization and the traditional Grey-Markov forecasting model.

1. Introduction

Grey system theory is suitable for partially unknown and poor information system modeling in a various fields, such as engineering [1,2], computer science [3], energy fuels [4,5], aviation [6], economy [7,8]. Duo to model's characteristics, Grey Model(1,1) (GM(1,1)), which stands for one-variable and first-order differential equation, mainly adapts to quasi-exponential and quasi-smooth data sequences. For the data sequence with the high volatility, the fitting results of GM(1,1) are relatively inaccurate, let alone the prediction values. On the other hand, Markov chain is suitable for long-term data sequences with large random fluctuations. Nowadays, considering most data sequence's characteristics of unitization and volatility, Grey-Markov forecasting model which combines grey forecasting model with markov chain is built to optimize the prediction of these data.

In recent years, researchers and scholars have drawn an increasing attention to the applications of Grey-Markov model in the fields of machinery [9], security [10], weather [11], traffic transportation [12,13], chemistry [14] etc. Meanwhile, there are also a great deal of research achievements in the improvement of the forecasting process of Grey-Markov model. Specifically, in Grey-Markov Forecasting Model, GM (1,1)'s background value and initial value was improved in [15]. The recurrence sequence was used in the GM(1,1) fitting data sequence to ensure the replacement of old and new information, and

* Corresponding author.
E-mail address: yejingjessie@163.com (J. Ye).

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Markov chain was used to modify the residual sequence [16]. Hsu et al. combined Fourier series and Markov state transition, known as Markov-Fourier grey model (MFGM), to predict the turning time of Taiwan weighted stock index (TAIEX) [17]. Grey-Markov model, Grey-Model with rolling mechanism, and singular spectrum analysis (SSA) were applied to forecast the consumption of conventional energy in India [18].

Nevertheless, it is extremely difficult to classify each state accurately due to the lack of information. And the growing interest in seeking simple and objective method in state division of Grey-Markov model has been heightened. In addition, whether it is possible to choose a reasonable data processing method and rationally divide the state of each object would directly affect the prediction results for original data sequence. Scholars have already used the Markov switching model [19], Markov state transition model [20], Markov model with neighborhood search algorithm [21] and the hidden Markov model [22,23] to try to optimize the state division. And these research achievements have enriched and deepened the applications of Grey-Markov Forecasting Model.

To seek a simple and objective method for states division, the central-point triangular whitenization weight function is introduced in this paper to make up the limitations of the traditional method's subjectivity. The central-point triangular whitenization weight function is a simple function with a central point and a state interval. To determine the central point, the average value of each interval can be adopted to express the maximum possibility of each state interval that makes the state division process more objective and practical.

Going back to the prediction of the traditional Grey-Markov model, GM(1,1) is regularly used to process original data, and then the state transition probability can be obtained by subjective experiences according to the limited information to modify forecasting results. This paper addresses the prediction process by using the GM(1,1) with background value optimization to model the raw data and then calculating state possibilities based on the central-point triangular whitenization weight function to solve the problem of state division. Thereby, Grey-Markov Forecasting Model based on background value optimization and the central-point triangular whitenization weight function is established.

The remaining paper is organized as follows. In Section 2, the basic model of GM(1,1) and Markov chain are presented, including the operations of modeling process. Based on the concepts in Section 2, the proposed new model for Grey-Markov forecasting model based on background value optimization and the central-point triangular whitenization weight function is shown in Section 3. Then, an illustrative example is used to demonstrate the feasibility and practicability of the proposed method in Section 4. Finally, Section 5 concludes this paper.

2. Modeling denotations

Grey-Markov forecasting model is a combination model which is based on Grey model and Markov chain. The modeling process of traditional GM(1,1) [24] and Markov chain are described as follows.

2.1. GM (1,1) modeling mechanism

Let $X^{(0)} = (x^{(0)}(1), x^{(0)}(2), ..., x^{(0)}(n))$ be original data sequence, $x^{(0)}(k) \geq 0 (k = 1, 2, \cdots, n)$.

In order to evaluate GM(1,1) forecasting reasonably, the first $n-m$ items are selected as modeling data, and the last m items $(x^{(0)}(n-m+1), x^{(0)}(n-m+2), \cdots, x^{(0)}(n-1), x^{(0)}(n))$ are regarded as forecasting data.

$X^{(1)}$ is the 1-AGO sequence of $X^{(0)}$,

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i) (k = 1, 2, \cdots, n-m) \tag{1}$$

$Z^{(1)}$ is the adjoining mean generated sequence of $X^{(1)}$,

$$z^{(1)}(k) = \frac{1}{2}\left(x^{(1)}(k) + x^{(1)}(k-1)\right)(k = 2, 3, \cdots, n-m) \tag{2}$$

$z^{(1)}(k)$ is the original background value.

The basic form of GM(1,1) is:

$$x^{(0)}(k) + az^{(1)}(k) = b \tag{3}$$

a is the development coefficient; a, b are the parameters of the differential equation.

Calculate the parameters, $\hat{a} = [a, b]^T = (B^T B)^{-1} B^T Y$

Among them, $B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix}$, $Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}$

The albino equation of the basic GM(1,1) is:

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b \tag{4}$$