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Research paper

Phase transition of a new lattice hydrodynamic model with consideration of on-ramp and off-ramp

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ABSTRACT

A new traffic lattice hydrodynamic model with consideration of on-ramp and off-ramp is proposed in this paper. The influence of on-ramp and off-ramp on the stability of the main road is uncovered by theoretical analysis and computer simulation. Through linear stability theory, the neutral stability condition of the new model is obtained and the results show that the unstable region in the phase diagram is enlarged by considering the on-ramp effect but shrunk with consideration of the off-ramp effect. The mKdV equation near the critical point is derived via nonlinear reductive perturbation method and the occurrence of traffic jamming transition can be described by the kink–antikink soliton solution of the mKdV equation. From the simulation results of space-time evolution of traffic density waves, it is shown that the on-ramp can worsen the traffic stability of the main road but off-ramp is positive in stabilizing the traffic flow of the main road.

1. Introduction

Traffic congestion has attracted considerable attention of scholars and engineers due to its significant impact on our daily life. In order to reveal the underlying mechanism of traffic congestion in detail, many traffic models have been proposed from different perspective. In general, we can classify these models into the macroscopic models and the microscopic models. In the microscopic modeling, each individual vehicle is regarded as a particle and the traffic is considered as a system of interacting particles driven far from equilibrium [1]. The macroscopic models treat the whole road traffic as compressible fluid formed by vehicles and uncover traffic properties by analyzing the relation among traffic flow, mean speed and traffic density [2]. It is found out that many actual traffic phenomena such as the stop-and-go traffic waves and traffic jamming transition can be reproduced in these models.

In recent years, the nonlinear analysis has been performed widely to study traffic jamming transition between the freely moving traffic and the jammed traffic. In 1995, Kurtze and Hong [3] derived the Korteweg–de Vries (KdV) equation from the macroscopic hydrodynamic model and showed that the traffic jam near the neutral stability curve can be described as soliton density waves. Later, Komatsu and Sasa [4] analyzed the traffic waves of the microscopic optimal velocity (OV) model and derived the modified Korteweg–de Vries (mKdV) equation to describe traffic jam near the critical point as kink–antikink

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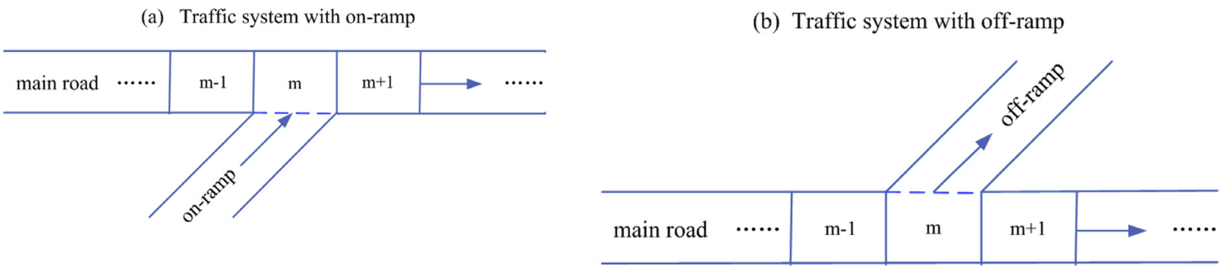

Fig. 1. The schematic of traffic system with on-ramp or off-ramp.

density waves. After that, the nonlinear analysis has been carried out for lots of extended macroscopic and microscopic traffic models.

In 1998, Nagatani [5] proposed the first lattice hydrodynamic model in view of the microscopic car-following theory on a single lane road. As the lattice hydrodynamic model possesses the merits of both the macroscopic hydrodynamic models and the microscopic car-following models, it has a simple structure and is very convenient for theoretical analysis and simulation. Subsequently, lots of extended works have been performed by taking various kinds of real traffic information into account. In these models, some are extended by considering the multiple lattices effect [6–8], some are modifications of the original model by introducing the density or current difference effect [9–14], and other works were conducted by taking driver's individual behavior effect into account [15–20]. Furthermore, due to the fact that most of the actual roads are made up of two lanes or more, many extended two-lane lattice hydrodynamic models were constructed [21–26].

In real traffic, the traffic systems with on-ramp or off-ramp are widespread and the influences of on-ramp or off-ramp on the traffic stability of the main road have been widely studied based on the cellular automata models [27–31]. Empirical observation of freeway traffic show that the phase transition from freely moving flow to traffic congestion occurs mostly at on-ramp bottleneck and the off-ramp usually can alleviate the seriousness of traffic congestion of the main road. But few researches have been conducted to reveal the effects of the on-ramp or off-ramp in lattice hydrodynamic model so far to our knowledge [32]. Therefore, a new lattice hydrodynamic model with consideration of on-ramp and off-ramp is proposed. The new model is introduced in the following section. Section 3 relates to the linear stability analysis of the new model. In Section 4, we deduce the mKdV equation to describe the properties of traffic jamming transition. Numerical simulations are carried out to validate the analytic results in Section 5 and a conclusion is given in Section 6.

2. The new model

The first lattice hydrodynamic model proposed by Nagatani [5] for single lane highway in 1998 is derived by discretizing the following continuum model:

$$\partial_t \rho + \partial_x(\rho v) = 0, \tag{1}$$

$$\partial_t(\rho v) = a\rho_0 V(\rho(x+\delta)) - a\rho v, \tag{2}$$

where a is the driver's sensitivity; ρ_0 denotes the average density; δ represents the average headway, and $\delta = 1/\rho_0$; V refers to the optimal velocity; $\rho(x+\delta)$ is the local density at position $x+\delta$ at time t, which equals the inverse of the headway $h(x, t)$ at position x , namely, $\rho(x+\delta) = 1/h(x,t)$. Eq. (1) is the continuity equation which relates the local density $\rho(x, t)$ of traffic flow to the local speed $v(x, t)$. Eq. (2) is the evolution equation which means that the variation of traffic current at position x is determined by the difference between the optimal current $\rho_0 V(\rho(x+\delta))$ and the actual current ρv.

The above model is further modified with dimensionless space x (let $\tilde{x} = x/\delta$, and $\tilde{x}$ is indicated as x hereafter) and can be rewritten in the following lattice version:

$$\partial_t \rho_m + \rho_0(\rho_m v_m - \rho_{m-1} v_{m-1}) = 0, \tag{3}$$

$$\rho_m(t+\tau) v_m(t+\tau) = \rho_0 V(\rho_{m+1}(t)), \tag{4}$$

where m denotes the lattice number; ρ_m and v_m represent the local density and local velocity of lattice m at time t, respectively; τ equals to the inverse of driver's sensitivity and it is called driver's delay time.

In real traffic, the on-ramp and off-ramp are great components in many traffic systems and they would affect the stability of the main road significantly. Fig. 1 shows the schematic of traffic system with on-ramp (a) or off-ramp (b). In Fig. 1(a), with the traffic flow of the on-ramp injects into the main road at lattice m, the traffic density of lattice m of the main road is bigger than that of lattice $m-1$. Therefore, the traffic incoming flow at lattice m can be defined as $\beta|\rho_0^2 V'(\rho_0)|(\rho_m - \rho_{m-1})$, where β denotes the on-ramp rate of lattice m and the constant $|\rho_0^2 V'(\rho_0)|$ is introduced to be dimensionless and $V'(\rho_0)$ denotes the first-order derivative of the optimal velocity V at ρ_0. Similarly, we can see from Fig. 1(b) that the traffic density