Commun Nonlinear Sci Numer Simulat 52 (2017) 44–51

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Research paper

Propagation of acoustic unipolar pulses and periodic waves in media with quadratic hysteretic nonlinearity and linear viscous dissipation

A. Radostin [a,b,*], V. Nazarov [a], S. Kiyashko [a]

[a] Institute of Applied Physics RAS, Nizhny Novgorod, Russia
[b] Department of Applied Mathematics, Nizhny Novgorod State Technical University, Nizhny Novgorod, Russia

ARTICLE INFO

Article history:
Received 10 August 2016
Revised 14 February 2017
Accepted 19 April 2017
Available online 20 April 2017

Keywords:
Hysteretic media
Nonlinear waves
Self-similar solution

ABSTRACT

Results of theoretical study and numeric simulation of propagation of plane acoustic unipolar pulses and periodic waves in media with quadratic hysteretic nonlinearity and linear viscous dissipation are presented. The exact analytical solution is derived for self-similar pulses. Numerical simulation of propagation of initially triangular pulses and harmonic waves was carried out using spectral method. Agreement between analytical and numeric calculations is discussed.

1. Introduction

The theory of wave processes in homogeneous media with elastic analytical (quadratic or cubic) nonlinearity is developed rather well [1–6]. Exact solutions to problems of wave propagation in media with quadratic nonlinearity and linear viscosity are obtaining by using Hopf–Cole substitution [1–4], reducing the Burgers equation to the linear diffusion equation. For the case of unipolar pulses the self-similar solution is also known [1,3], its shape depends on ratio of parameters characterizing nonlinearity and dissipation. In media with cubic nonlinearity and linear dissipation the numeric technique based on the spectral approach is usually employed [6].

Considerable recent attention of researchers in the area of acoustics has been focused on micro-inhomogeneous (or mesoscopic) media, comprising most of rocks, polycrystalline metals, some artificial materials etc. [2,4,7,8]. Nonlinear properties of these media essentially differ from ones inherent to gases, liquids, and homogeneous solids. Consequently even simplified equations, analogs to the Burgers equation for homogeneous media, have more sophisticated form that restricts the possibility of obtaining of their exact solutions.

One of the most intriguing problems in this area is study of acoustic wave propagation in media with hysteretic nonlinearity [4,7–13]. As is shown in [9–13] solving problems of this sort in "pure nonlinear case" (neglecting different linear factors) presents no special problems. For instance a perturbation of initially arbitrary shape transforms into wave or pulse of triangular shape without formation of discontinuity in the wave form, as opposed of the case of quadratic nonlinearity, however, the time-derivative of the profile becomes discontinuous both in maximum and minimum of the wave. Thus finding an analytical solution for the wave profile in media with hysteretic nonlinearity and linear dissipation, eliminating the

* Corresponding author.
E-mail address: radostin@hydro.appl.sci-nnov.ru (A. Radostin).

http://dx.doi.org/10.1016/j.cnsns.2017.04.016
1007-5704/

discontinuity of the time-derivative, is of interest for development of theory of nonlinear wave propagation in media with hysteretic nonlinearity.

An evolution of waves in such media is only studied by using of perturbation methods or numerically to date. In [10] an approximate solution for the wave profile in media with hysteretic nonlinearity and linear dissipation was derived by expansion of function describing wave profile in terms of perturbation in the vicinity of maximum of the wave by using the ratio of parameters characterizing nonlinearity and dissipation as small parameter. In [14] this approach was generalized and employed for the case of Rayleigh scattering. In [15] this problem was considered by using spectral approach and successive approximation expansion. It is also worth noting paper [16] where purely numeric pseudo-spectral approach was used.

This paper deals with analytical and numeric study of plane acoustic unipolar pulses and periodic waves' propagation in media with quadratic hysteretic nonlinearity and linear viscous dissipation. An exact solution for self-similar pulses propagation is derived. The numeric simulation was performed on the basis of spectral approach proposed in [17] for bimodular media with linear dissipation.

2. Wave equation for media with hysteretic nonlinearity and linear dissipation

The dynamic equation of state for a media with hysteretic nonlinearity and linear viscous dissipation has form [4]:

$$\sigma(\varepsilon,\dot{\varepsilon}) = E[\varepsilon - f(\varepsilon,\mathrm{sgn}\dot{\varepsilon})] + \alpha\rho\dot{\varepsilon}, \tag{1}$$

where σ, ε and $\dot{\varepsilon}$ are stress, strain, and the rate of strain, respectively; E is Young's modulus; $f(\varepsilon,\mathrm{sgn}\dot{\varepsilon})$ is nonlinear (here hysteretic) function; $|f(\varepsilon,\mathrm{sgn}\dot{\varepsilon})| << |\varepsilon| << 1$, $|df(\varepsilon,\mathrm{sgn}\dot{\varepsilon})/d\varepsilon| << 1$; α a is viscosity coefficient, ρ is density.

A propagation of unipolar strain pulses we will consider in the media with the elastic hysteresis [4,18], when $f(\varepsilon = 0,\mathrm{sgn}\dot{\varepsilon}) = 0$. In the case of $\sigma \geq 0$, $\varepsilon \geq 0$, the hysteretic function has form [4,12,19]:

$$f(\varepsilon,\mathrm{sgn}\dot{\varepsilon}) = \frac{\gamma}{2}\begin{cases}\varepsilon^2, & \dot{\varepsilon} > 0;\\ 2\varepsilon_m\varepsilon - \varepsilon^2, & \dot{\varepsilon} < 0,\end{cases} \tag{2}$$

where γ is parameter of hysteretic nonlinearity ($\gamma > 0$), $\varepsilon_m = \varepsilon_m(x)$ is strain amplitude, $\gamma\varepsilon_m < <1$, $\gamma > >1$.

A propagation of periodical (bipolar) strain waves we will consider in the media with the inelastic (or plastic) hysteresis [4,18], when $f(\varepsilon = 0,\mathrm{sgn}\dot{\varepsilon}) \neq 0$. In this case the hysteretic function has form [4,12,19]:

$$f(\varepsilon,\mathrm{sgn}\dot{\varepsilon}) = \gamma\left(\varepsilon_m\varepsilon + \frac{1}{2}[\varepsilon^2 - \varepsilon_m^2]\mathrm{sgn}\dot{\varepsilon}\right). \tag{3}$$

Note that hysteretic function in the form (3) coincides with Rayleigh's magnetic hysteresis (with the accuracy of notations) [20–21].

Both hysteretic functions originally were proposed phenomenologically on the basis of analysis of experimental data [19], or they can be derived as cumulative response of ensemble of hysteretic mechanical elements (so-called Preisach–Mayergoyz space formalism [7,8]) as in [11,12]. The elastic type of hysteresis can also be considered as approximation of dislocation hysteresis proposed by Granato and Lücke with nonlinear restoring force [4].

Equation of state (1) together with the equation of motion $\rho U''_{tt} = \sigma'_x(\varepsilon,\dot{\varepsilon})$ [1–4], determine wave processes in nonlinear media with linear dissipation ($\varepsilon = \partial U/\partial x$, U is the displacement). Substitution of Eq. (1) into the equation of motion and using the method of a slowly varying wave profile [1–4] yield the following equation for strain wave propagating in the positive direction of the x axis:

$$\frac{\partial\varepsilon}{\partial x} = -\frac{1}{2C}\frac{\partial f(\varepsilon,\mathrm{sgn}\dot{\varepsilon})}{\partial t_1} + \frac{\alpha}{2C^3}\frac{\partial^2\varepsilon}{\partial t_1^2}. \tag{4}$$

where $t_1 = t - x/C$, $C = (E/\rho)^{1/2}$. Note that changes in waveform is supposed to be minor on the distance of wavelength $\lambda = 2\pi C/\omega$, where ω is frequency, hence, this equation is valid when $\pi\alpha\omega/C^2 < <1$, $|f(\varepsilon,\mathrm{sgn}\dot{\varepsilon})| << |\varepsilon|$ [1,2].

3. Self-similar solution for unipolar pulses in media with elastic hysteresis

The dimensionless form of an evolution Eq. (4) describing propagation of positive unipolar pulses has form:

$$\frac{\partial e}{\partial z} + e\frac{\partial e}{\partial\theta} = \frac{1}{\Gamma}\frac{\partial^2 e}{\partial\theta^2}, \ \frac{\partial e}{\partial\theta} > 0, \tag{5}$$

$$\frac{\partial e}{\partial z} + [e_m(z) - e]\frac{\partial e}{\partial\theta} = \frac{1}{\Gamma}\frac{\partial^2 e}{\partial\theta^2}, \ \frac{\partial e}{\partial\theta} < 0. \tag{6}$$

where $e = \varepsilon/\varepsilon_0 \geq 0$, $e_m(z) = \varepsilon_m/\varepsilon_0$, $\varepsilon_0 = \varepsilon_{\max}(x = 0)$, $z = \gamma\varepsilon_0 x/(CT_0)$, $\theta = t_1/T_0$, T_0 is the initial characteristic duration of the pulse, $\Gamma = \frac{2\gamma\varepsilon_0 C^2 T_0}{\alpha}$ is the Gol'dberg number for hysteretic media [3,4].