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Negative mobility of a Brownian particle: Strong damping regime

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ABSTRACT

We study impact of inertia on directed transport of a Brownian particle under non-equilibrium conditions: the particle moves in a one-dimensional periodic and *symmetric* potential, is driven by both an unbiased time-periodic force and a constant force, and is coupled to a thermostat of temperature T. Within selected parameter regimes this system exhibits negative mobility, which means that the particle moves in the direction opposite to the direction of the constant force. It is known that in such a setup the inertial term is *essential* for the emergence of negative mobility and it cannot be detected in the limiting case of overdamped dynamics. We analyse inertial effects and show that negative mobility can be observed even in the strong damping regime. We determine the *optimal* dimensionless mass for the presence of negative mobility and reveal three mechanisms standing behind this anomaly: deterministic chaotic, thermal noise induced and deterministic non-chaotic. The last origin has never been reported. It may provide guidance to the possibility of observation of negative mobility for strongly damped dynamics which is of fundamental importance from the point of view of biological systems, all of which *in situ* operate in fluctuating environments.

1. Introduction

If a system is at thermal equilibrium, its reaction to a weak external static force is so that the response is in the same direction of this applied force, towards a new equilibrium. E.g., when a constant force acts on a particle, it moves in the same direction of the force. If it is an electronic (electrical) device, the current-voltage dependence exhibits the similar properties, i.e. if the voltage increases the current increases. The ohmic characteristics is an example of it. It is what we call the normal transport. This restriction is no longer valid under nonequilibrium conditions when already an unperturbed system may exhibit a current due to the ratchet effect [1]. Another example is the seemingly paradoxical situation of the negative mobility phenomenon when the system response is opposite to the applied constant force [2]. Such anomalous transport behaviour was predicted theoretically in 2007 in a system consisting of an *inertial* Brownian particle moving in a one-dimensional periodic symmetric potential [3]. Within a year of this discovery, negative mobility was confirmed experimentally in the experiment involving determination of current-voltage characteristics of the microwaved-driven Josephson junction [4]. Yet further examples of this phenomenon have been described theoretically in companionship of coloured noise [5], white Poissonian noise [6], dichotomous process [7] and for Brownian motion with presence of time-delayed feedback

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[8,9], non-uniform space-dependent damping [10] and potential phase modulation [11]. Other illustrations include a vibrational motor [12], two coupled resistively shunted Josephson junctions [13,14], active Janus particles in a corrugated channel [15], entropic electrokinetics [16] as well as nonlinear response of inertial tracers in steady laminar flows [17].

Modelling systems and understanding their generic properties discloses which components of the setup are crucial and which elements may be sub-relevant. For instance, transport in the micro-world is strongly influenced by fluctuations and random perturbations. In some systems, like biological cells [18], they can even play a dominant role and a typical situation is that motion of particles is strongly damped. This fact justifies the use of an overdamped dynamics for which the particle inertial term $M\ddot{x}$ can be formally neglected in comparison to the dissipation term $\Gamma\dot{x}$ (M is the particle mass, Γ is the friction coefficient and dot denotes a differentiation with respect to time t). Omission of the inertial term enormously simplifies the modelling and in many cases allows for an analytical solutions of the corresponding Fokker–Planck equation. However, properties and features which are allowed to occur in systems with inertia can completely disappear when the inertial term is put to zero. Certainly a more correct approach in such a situation is to include the inertial term and use a technique of mathematical sequences of smaller and smaller dimensionless mass. Our main objective is to investigate impact of inertia on negative mobility of a Brownian particle moving in one-dimensional periodic systems. It is known that in such setups the inertial term is one of the key ingredients for the occurrence of this form of anomalous transport [19] and negative mobility is absent for the overdamped dynamics when $M\ddot{x} = 0$. We address the question whether it is still possible to observe the negative mobility phenomenon in strongly dissipative systems. In doing so, we first formulate the model and introduce the quantities of interest. Then we investigate the general transport behaviour as a function of model parameters and detect the optimal dimensionless mass for the presence of negative mobility. In the next part we demonstrate three mechanisms responsible for the emergence of this anomalous transport phenomenon: deterministic chaotic, thermal noise induced and deterministic non-chaotic. Finally, we discuss impact of inertia on the directed long time particle velocity and provide some conclusions.

2. Model

The model of a Brownian particle moving in a one-dimensional periodic landscape has been already well established in the literature [20]. It has been used to explore a wide range of phenomena including ratchet effects [21–23], noise induced transport [24], the negative mobility [3], the enhancement of transport [25], diffusion phenomena [26,27] and Gaussian white noise as a resource for work extraction [28]. Here, we consider exactly the same model as in [3]: a classical inertial Brownian particle of mass M, which moves in a spatially periodic potential $U(x) = U(x + L)$ of period L and is subjected to both an unbiased time-periodic force $A\cos(\omega t)$ of amplitude A and angular frequency Ω and an external static force F. Dynamics of such a particle is described by the following Langevin equation [3]

$$M\ddot{x} + \Gamma\dot{x} = -U'(x) + A\cos(\Omega t) + F + \sqrt{2\Gamma k_B T}\,\xi(t), \tag{1}$$

where prime denotes a differentiation with respect to the particle coordinate x. Thermal fluctuations due to the coupling of the particle with the thermal bath of temperature T are modelled by Gaussian white noise of zero mean and unity intensity, namely

$$\langle \xi(t) \rangle = 0, \quad \langle \xi(t)\xi(s) \rangle = \delta(t - s). \tag{2}$$

The noise intensity factor $2\Gamma k_B T$ (where k_B is the Boltzmann constant) follows from the fluctuation-dissipation theorem [29] and ensures the canonical equilibrium Gibbs state when $A = 0$ and $F = 0$. The potential $U(x)$ is assumed to be in a *symmetric* form with the period L and the barrier height $2\Delta U$, namely,

$$U(x) = \Delta U \sin\left(\frac{2\pi}{L}x\right). \tag{3}$$

There exists a wealth of physical systems that can be described by the Langevin equation (1). Important cases that come to mind are the semiclassical dynamics of a phase difference across a resistively and capacitively shunted Josephson junction [30] and a cold atom moving in an optical lattice [1,31]. Other examples include superionic conductors [32], dipoles rotating in external field [33], charge density waves [34] and adatoms on a periodic surface [35].

2.1. Scaling and dimensionless Langevin equation

Since only relations between scales of length, time and energy are relevant for the observed phenomena, not their absolute values, we next formulate the above presented equation of motion in its dimensionless form. This can be achieved in several ways [36]. Because investigation of impact of the particle inertia on the system dynamics is our main goal, in the present consideration we propose the use of the following scales as the characteristic units of length and time [36]

$$\hat{x} = \frac{x}{L}, \quad \hat{t} = \frac{t}{\tau_0}, \quad \tau_0 = \frac{\Gamma L^2}{\Delta U}. \tag{4}$$

Under such a procedure the Langevin equation (1) takes the dimensionless form

$$m\ddot{\hat{x}} + \dot{\hat{x}} = -\hat{U}'(\hat{x}) + a\cos(\omega\hat{t}) + f + \sqrt{2D}\hat{\xi}(\hat{t}). \tag{5}$$