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Research paper

Fractal structures in centrifugal flywheel governor system

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ABSTRACT

The global structure of nonlinear response of mechanical centrifugal governor, forming in two-dimensional parameter space, is studied in this paper. By using three kinds of phases, we describe how responses of periodicity, quasi-periodicity and chaos organize some self-similarity structures with parameters varying. For several parameter combinations, the regular vibration shows fractal characteristic, that is, the comb-shaped self-similarity structure is generated by alternating periodic response with intermittent chaos, and Arnold's tongues embedded in quasi-periodic response are organized according to Stern–Brocot tree. In particular, a new type of mixed-mode oscillations (MMOs) is found in the periodic response. These unique structures reveal the natural connection of various responses between part and part, part and the whole in parameter space based on self-similarity of fractal. Meanwhile, the remarkable and unexpected results are to contribute a valid dynamic reference for practical applications with respect to mechanical centrifugal governor.

1. Introduction

Centrifugal governor is a kind of mechanical device, which can take command of engine speed automatically. Specially, it plays a key role in rotating machinery, for instance, diesel engine, petrol engine, electronic centrifugal and water turbine. Since centrifugal governor is always used widely in engineering, its dynamic characteristics have attracted more interest in the last 20 years or so. It has been turned out that the rotating behavior of centrifugal governor displays complicated and abundant dynamics experimentally, numerically, and analytically. Ge et al. [1–4] surveyed the dynamic behaviors of various centrifugal governor, and found the routes from period-doubling bifurcation and Hopf bifurcation to chaos. In Refs. [5,6], Sotomayor et al. obtained sufficient conditions of codimension one and two Hopf bifurcation appearing in a Watt governor by means of analysis method. In addition, Chu and Zhang [7–9] also demonstrated the process from Hopf bifurcation to chaos by taking account of codimension one bifurcation diagrams and a sequence of Poincaré section under different sets of system parameters. Recently, Wen and his cooperators have extended work related to the anti-controlling Hopf bifurcation in Ref. [10].

As already mentioned, although nonlinear responses of centrifugal governor were already investigated abundantly, they were handled most frequently through analyzing the dynamics observed along specific one-dimensional path or several

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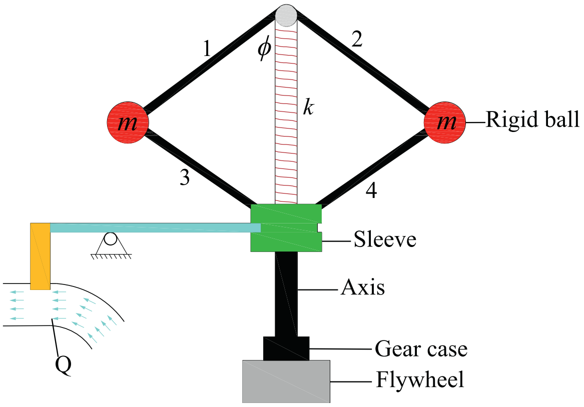

Fig. 1. physical model of the mechanical centrifugal flywheel governor.

scattered one-parameter sections of parameter spaces that are normally of very high-dimension. Codimension one bifurcation serves as a significant tool to understand the dynamic behavior of centrifugal governor varying with parameters. It is common to only achieve some local information, however, some important dynamics information may be neglected. Naturally, from a control and optimal design point of view, it is necessary to explore the global dynamics information. In essence, it is the basic problem to solve all sorts of nonlinear responses. It remains the salient problem in the field. Lately, investigating the dynamics of nonlinear system in two-dimensional parameter space has been a focus. Thus, a large number of far-reaching regular structures about two-dimensional parameter space are born. For instance, periodic and chaotic hubs [11–18] namely infinite-hierarchy nested spirals, a sort of “eye” of chaos [24] emerging as the accumulation limit of a doubling cascade, small regular shrimp-shaped islands [19–23], Farey tree [25,26], as a kind of tree-shaped hierarchical organization. Surprisingly, these novel structures in a way of self-similarity clearly expose the unsuspected law of nonlinear system by organizing various cases of nonlinear responses.

The aim of this paper is to investigate the oscillation structure arising from various vibration responses existing in mechanical centrifugal governor in two-dimensional parameter space. Numerical method is adopted to present some high-resolution phase diagrams, which are in determining the extent in control parameter space of the various response regimes, and understanding the transitions between periodic, quasi-periodic and chaotic modes. Meanwhile, the wider range about parameter choice of the optimal design and control of mechanical centrifugal governor can be provided.

2. The dynamical model of centrifugal governor

The sketch of the mechanical centrifugal governor system [27] is shown in Fig. 1. The engine drives the flywheel to rotate with angle velocity w, the flywheel is linked to the axis through a gear case, and the axis rotates with the angle velocity nw. Each of the axis head is hinged two rods with the length l respectively. The other heads of rod 1 and rod 2 are also attached to a ball with mass m, and rod 3 and rod 4 are connected to the sleeve. In addition, a spring with elastic coefficient k is installed on the axis. The centrifugal governor system can adjust the amount of fuel Q, and control the flywheel with constant angle velocity w_0 rotating. If the speed of the engine slows down, which brings about the centrifugal force acting on the rigid ball decrease, the control valve will open wider to provide more fuel for the engine. If there is more fuel is supplied, the engine then will be driven faster, the centrifugal force acting on the rigid ball increases and the control valve turns down, till the equilibrium of the whole system is once again reached.

In order to conveniently analyze, the following assumptions are commonly accepted in the open literature: (1) neglect the mass of the sleeve and rods; (2) the viscous fiction coefficient in rod bearing of fly-ball is represent by constant coefficient c.

The kinetic energy T and potential energy V of the system are given by

$$T = 2\times\left\{\frac{1}{2}m[(l\sin\phi)^2(nw)^2+l^2\dot{\phi}^2]\right\} = m(nw)^2(l\sin\phi)^2+ml\dot{\phi}^2, \tag{2.1}$$

$$V = 2kl^2(1-\cos\phi)^2+2mgl(1-\cos\phi), \tag{2.2}$$

where ϕ is the angle between the rotational axis and the rods. Lagrange’s function L is easy to obtain from Eq. (2.1) and Eq. (2.2),

$$L = T-V = m(nw)^2(l\sin\phi)^2+ml^2\dot{\phi}^2-2kl^2(1-\cos\phi)^2-2mgl(1-\cos\dot{\phi}), \tag{2.3}$$

from Eq. (2.3), the dynamic equations can be expressed,

$$2ml^2\ddot{\phi}-2ml^2(n\omega)^2\sin\phi\cos\phi+4kl^2\times(1-\cos\phi)\sin\phi+2mgl\sin\phi = -c\dot{\phi}. \tag{2.4}$$