Energy Conversion and Management 150 (2017) 295–303

Contents lists available at ScienceDirect

Energy Conversion and Management

journal homepage: www.elsevier.com/locate/enconman

Modeling the dynamic and thermodynamic operation of Stirling engines by means of an equivalent electrical circuit

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ARTICLE INFO

Keywords:
Electrical analogy
Free-piston Stirling engine
RE-1000
Conservation equations
Equivalent circuit
Laplace transform

ABSTRACT

The Stirling engines are inherently efficient; their thermodynamic cycles reach the Carnot efficiency. These technologies are suitable to operate under any low temperature difference between the hot and the cold sources. For these reasons, these engines can be considered as reliable power conversion systems to promote the conversion of low-grade waste heat generated by industrial plants. The need of a model to predict the behavior of these engines is of primary importance. Nevertheless, a great difficulty is encountered in developing such a model since it is not simple to take into account coupled thermodynamic and dynamic effects. This is the main reason why several models make use of electrical analogies to describe Stirling engines (in particular, free-piston machines): by assuming the pressure equivalent to a voltage and the flow rate to an electrical current, a coupled dynamic-thermodynamic analysis of the engine can be performed. In this paper, an electrical circuit whose behavior is equivalent to that of the engine is derived from the electrical analogy theory. To this aim, we propose an electrical analogy model based on the three conservation laws (mass, momentum and energy). Since limited experimental information is available in the open literature, the results obtained with the proposed model are compared with the experimental data collected at the NASA Lewis Research center for a free-piston Stirling engine i.e., the RE-1000 engine.

1. Introduction

Many industrial plants are energy intensive. This aspect has brought many engineers to investigate solutions to enhance the energy efficiency; such an improvement cannot be reached by simply reducing the energy consumption, as the latter is usually proportional to the production of output goods (e.g., the aluminum production in a smelter is linearly proportional to its energy consumption). Therefore, recovering thermal wastes, which are usually rejected to the environment, is of primary importance. They can be used for heating purposes (e.g., district heating) or converted into useful work, by means of an appropriate conversion technology (for instance, by using Organic Rankine Cycle engines, thermoelectric modules or Stirling engines) [1].

Amongst all these technologies, Stirling engines have gained the interest of the scientific community [2,3]. Several reasons justify this concern. An analysis of the ideal Stirling cycle shows that it reaches the Carnot-efficiency limit, since it undergoes two isothermal and two isochoric processes [4,5]. Another important aspect of Stirling engines is their ability of converting any heat source into useful work. Several patents of Stirling engines powered by solar energy [6], as well as by fossil fuels [2,3,7], are available in the open literature. Moreover, some of them are able to work under low temperature differential [8–10]. For instance, Formosa et al. [11,12] assembled a Stirling engine for which the hot source is at 150 °C and the cold one is at 25 °C. This characteristic makes these systems attractive for low–potential energy recovery purposes [10]. Furthermore, the Stirling engine technology is reliable at relatively low cost [2,3,11,10] and satisfies several industrial safety requirements, as the working fluid is not hazardous for metallurgic plants (i.e., the working fluid may be a gas, such as air, helium or nitrogen).

The need of trustworthy models able to predict the behavior of these engines is mandatory. According to the most recent reviews [10], two modeling approaches are available in the open literature: those based on the heat transfer analysis and those based on the electrical analogy theory.

The well known Schmidt's analysis [13] falls into the heat-transfer category model; it describes the thermodynamics of the engine (expansion and compression chambers, heat exchangers), but it does not take into account the mechanical dynamics (i.e., the power piston and the displacer are not modeled). Thus, it is important to couple the

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http://dx.doi.org/10.1016/j.enconman.2017.08.027
Received 5 April 2017; Received in revised form 11 July 2017; Accepted 9 August 2017

Nomenclature

Greek Letters

α	dimensional coefficient
γ	ratio of heat capacities
κ	friction factor [1/s]
μ	viscosity [kg/s m]
ρ	density [kg/m^3]
τ	time [s]
ε	regenerator porosity

Symbols

$\overline{R}$	universal gas constant [8.314 J/mol K]
$\dot{V}$	volumetric flow rate (Laplace domain) [m^3/s]
$\dot{v}$	volumetric flow rate (time domain) [m^3/s]
$\dot{W}$	power [W]
$\mathscr{L}$	Laplace-transform operator
$\vec{g}$	gravitational acceleration
$\vec{q}$	heat flux [W/m^2]
$\vec{u}$	velocity [m/s]
A	surface area [m^2]
Pr	Prandtl number
R	equivalent resistor [kg/s m^4]
Re	Reynolds number
s	Laplace variable
T	temperature (Laplace domain) [K]
t	temperature (time domain) [K]
V_r	regenerator volume [m^3]
x	spatial coordinate [m]

Subscripts

c	compression chamber
d	displacer
Be	Beale number
C	equivalent capacitor [m^4 s/kg]
c_f	friction factor coefficient
c_p	specific heat [J/kg K]
D	diameter [m]
d_h	hydraulic diameter [m]
f	frequency [Hz]
F_d	damping force [N]
g	source term (time domain)
h	convective heat transfer coefficient $[W/m^2K]$
k	thermal conductivity [W/m K]
L	equivalent inductor [kg/m^4]
m	mass [kg]
n	exponential coefficient ($F_d = \alpha\dot{x}_p^n$)
Nu	Nusselt number
P	pressure (Laplace domain) [Pa]
p	pressure (time domain) [Pa]
e	expansion chamber
h	heater
HT	heat exchanger [m^3]
k	cooler
p	power piston
r	regenerator
s	metallic matrix
w	wall
max	maximum value
rms	root mean square

Other symbols

$\overline{\psi}$	mean variable
ψ'	fluctuating variable

aforementioned analysis with the Newton's laws of the moving parts. This approach has been firstly proposed by Urieli and Berchowitz [8], who studied several units; however, they observed noticeable errors between the predictions and the experimental data [8]. For instance, their model overpredicts the power output of a GPU-3 (General Motors Ground Power Unit) system with an error of at least 69%.

The second kind of model involves the use of the electrical analogy theory. The aim of this approach is to establish an analogy between the current with the volumetric flow rate and the voltage differentials with pressure variations. Such theory has been proposed by Swift [14] in order to predict the behavior of thermoacoustic systems. Moreover, the electrical analogy theory can be used to obtain an equivalent electrical network for free-piston systems. The electrical analogy theory [14] has been extensively validated in the literature. In particular, it is suitable to predict the behavior of thermoacoustic Stirling refrigerators. Huang and Chuang have used it to develop a linear model for an orifice pulse tube refrigerator [15]. Bailly and Nika have evaluated the equivalent electrical circuit of a double inlet miniature pulse tube refrigerator [16,17]. Similar works have been conducted, amongst others, by Iwase et al. [18] and by Tan and Dang [19].

Despite its application that is limited to thermoacoustic devices, the electrical analogy theory is interesting for Stirling engines too. Effectively, the transformations that the working gas undergoes in a thermoacoustic engine (both heat engines and refrigerators) are equivalent to those of a conventional Stirling system. However, it cannot be used to describe a conventional Stirling one because in thermoacoustic machines, there are no moving pistons. Thus, in order to describe a Stirling engine, the thermoacoustic approach can be used only if it is coupled to an appropriate dynamical analysis. Formosa et al. [11] proposed such a model; according to their theoretical approach, which has been extended by Féniès et al. [12], the dynamical effects are analyzed by means of the Newton's equation applied to the moving parts (following the approach given by Urieli and Berchowitz) [8]. They have applied the continuity and the momentum conservation equations to evaluate the gas pressure and velocity in the expansion and compression chambers, in the heater, in the cooler and in the regenerator [11,12].

In this paper, we present a theoretical approach that can be used to predict the behavior of free-piston Stirling engines. We propose to extend the Formosa et al.'s modeling approach by introducing the energy equations for the heat exchangers and the regenerator. In order to do that, the thermoacoustic approach of Huang et al. [15] is considered. It is noteworthy to highlight that the analysis of the energy equation has the only effect of introducing a source term, while the resistors, the capacitor and the inductor associated to the mass and the momentum conservation equations remain unchanged [14,20]. A detailed description of the model is presented in Section 2; thereafter, a comparison of the predictions with the data collected for a free piston Stirling engine [21] is outlined in Section 3. A parametric study is performed to investigate the influence of main design parameters, such as the mean gas pressure, the hot and cold gas temperatures in the heat exchangers. Finally, the predicted behavior of the engine is used to compute the output power.