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Small-signal model for frequency analysis of thermoelectric systems

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ABSTRACT

We show how small-signal analysis, a standard method in electrical engineering, may be applied to thermoelectric device performance measurement by extending a dc model to the dynamical regime. We thus provide a physical ground to *ad-hoc* models used to interpret impedance spectroscopy of thermoelectric elements from an electrical circuit equivalent for thermoelectric systems in the frequency domain. We particularly stress the importance of the finite thermal impedance of the thermal contacts between the thermoelectric system and the thermal reservoirs in the derivation of such models. The expression for the characteristic angular frequency of the thermoelectric system we obtain is a generalization of the expressions derived in previous studies. In particular, it allows to envisage impedance spectroscopy measurements beyond the restrictive case of adiabatic boundary conditions often difficult to achieve experimentally, and hence *in-situ* characterization of thermoelectric generators.

1. Introduction

Because of their small energy conversion efficiency thermoelectric solutions are typically less advantageous compared to other power-production technologies over a wide range of temperatures [1]. However, as thermoelectric systems do not require moving parts to operate, their reliability is higher than those of standard heat engines and, as thermoelectric energy conversion is essentially electronic in nature, thermoelectricity provides a convenient way to directly convert waste heat into electric power. So, in addition to the great efforts made for the improvement of device and materials performance, focusing especially on nanoscale structures [2,3], much work is also devoted to finding innovative solutions for the integration of thermoelectric devices in combined generation and refrigeration cycles [4–7]. For these latter schemes, it is obviously crucial to obtain reliable measurements of the overall efficiency to assess their technological and economic interest [8,9], but one of the challenges to be met is to properly and accurately determine the performance of the thermoelectric device itself, which can be related to the theoretical maximum efficiency of a thermoelectric generator working between two thermal reservoirs at temperatures T_h and T_c respectively, with $T_c < T_h$ [10,11]:

$$\eta_{\max} = \eta_C \frac{\sqrt{1+ZT}-1}{\sqrt{1+ZT}+T_c/T_h} \tag{1}$$

with $\eta_C = 1-T_c/T_h$ being the Carnot efficiency, and ZT the figure of merit being given by

$$ZT = \frac{\alpha^2\overline{T}}{RK_0} \tag{2}$$

where $\overline{T}$ is the average working temperature of the system, R is the electrical resistance of the module, K_0 is its thermal conductance at vanishing electrical current, and α is the global Seebeck coefficient characterizing the thermoelectric coupling between the electrical current and the heat flux across the legs of the device.

So thermoelectric performance boils down to the determination of a single quantity: ZT, but its accurate evaluation is far from being straightforward, and various approaches may apply: One may measure α,R, and K_0, separately and then compute ZT using Eq. (2). This method however proves quite inaccurate without great experimental care as each measurement error for each parameter contributes to the cumulated global error on the resulting value of ZT [12–15]. To overcome the unavoidable drawbacks of multiple measurements, Harman suggested that ZT might be determined by only measuring, under adiabatic conditions, the voltage across the sample resulting from alternating current; this technique is known as the transient Harman method [16]. Lisker later extended the idea of a single parameter measurement for different conditions: He proposed to measure electrical conductivity under adiabatic and isothermal conditions or, equivalently, to measure thermal conductivity under vanishing electrical current and short-circuit conditions [17]. Min and Row suggested that ZT might also be evaluated by measuring the temperature difference across the sample for short-circuit and open-circuit conditions if the incoming heat flows

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remain constant during the experiment [18].

Recently, the Harman technique has been further improved using impedance spectroscopy analysis to lower uncertainties on ZT [19]. With this technique, the frequency dependence of the voltage-current ratio across the system is analyzed and then compared to theoretical models. Besides the determination of ZT, it is also possible to gain additional information on the material's thermoelectric properties [20–24]. To evaluate the different characteristics of the sample, measurements are fitted to an equivalent electrical model. Amongst the different proposed equivalent models, the simplest one corresponds to an RC electrical circuit [20]. This latter was obtained empirically from measurements. In this article, we show how to associate this ac model with existing dc models for thermoelectric generators in order to interpret impedance spectroscopy measurements of thermoelectric modules explicitly accounting for their non-ideal coupling to heat source and sink. Indeed the presence of the finite thermal conductance of heat exchangers influences greatly the performance of thermoelectric devices, as shown, e.g., in optimization and performance improvement studies involving the thermal coupling to the heat sink in particular [25,26]. The article is organized as follows. In Section 2, we first present the dc model for thermoelectric generators including non-ideal thermal contacts. We then extend this model to the frequency domain using small-signal modeling. In Section 3, we apply numerically our small-signal analysis to a commercial device and we discuss our approach and compare our results to the literature stressing similarities and discrepancies with other models.

2. Model

2.1. Classical model with non-ideal thermal contacts

We consider a thermoelectric generator connected through non-ideal thermal contacts to two thermal reservoirs at constant temperatures T_h and T_c respectively. The wording "thermal contacts" implies all the parts of the actual system that conduct the heat flux including the heat exchangers, heat conductive paste, ceramic layers, and copper stripes. With no loss of generality, we then assume that the thermal contacts thus defined are characterized by a finite thermal conductance K_{hot} (resp. K_{cold}) on the hot (resp. cold) side. The electrical and thermal properties of this system in the dc regime are given by [27]:

$$V = \alpha\Delta T' - RI, \tag{3}$$

$$I_Q = \alpha\overline{T}I + K_0\Delta T' \tag{4}$$

where V and $\Delta T'$ are respectively the voltage and the temperature difference across the generator and I and I_Q are respectively the electrical current and the thermal current flowing through the device. To obtain the above equations, the effect of Joule heating is neglected and the thermal current is assumed to be constant along the device even if, actually, it slightly varies due to the energy conversion process. This assumption is quite reasonable in the generator regime under working conditions often met in practice when the temperature difference across the device, $\Delta T'$, is usually much smaller than the average temperature across the device $\overline{T}$, which is the case during, e.g., characterization at ambient temperature. It also greatly simplifies the model as I_Q is then only composed of two distinct contributions: A convective heat current $\alpha\overline{T}I$ associated with the global displacement of the electrons [28,29] and a conductive heat current $K_0\Delta T'$ associated with heat leaks. Note that the term *electric convection of heat*, in opposition to traditional heat conduction, has been coined in 1856 by Kelvin in Ref. [28]. Furthermore, we assume that the three thermoelectric parameters R,K_0 and α are constant and that the system exchanges heat only with the thermal reservoirs. The global system is described in Fig. 1. For completeness, the thermal capacitances of the thermoelectric generator, C_{th}, and of the thermal contacts, C_{hot} and C_{cold}, are also displayed even if they have no influence in the dc regime as they are then equivalent to a thermal open-circuit.

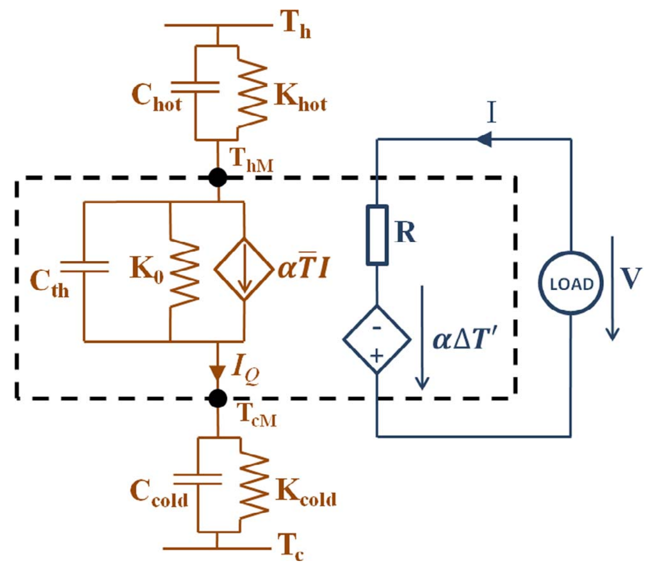

Fig. 1. Schematic representation of the thermoelectric converter with non ideal thermal contacts.

Even if it might be suggested by its form, Eq. (3) does not correspond to a genuine Thévenin modeling of the thermoelectric generator since the electromotive force $\alpha\Delta T'$ depends on the electrical current I [27]. This point is stressed by the use of a controlled voltage source rather than an ideal voltage source in Fig. 1. This dependence is due to the combined effects of the convective heat current and of the non-ideal thermal contact conductances: Because of the thermal contacts, the temperature difference $\Delta T'$ across the generator is different from the temperature difference between the thermal reservoirs ΔT, the latter being constant. Since we assume constant heat current I_Q across the device, the thermal circuit might be seen as a thermal equivalent of a voltage divider and one then gets:

$$\Delta T' = T_{hM} - T_{cM} = \Delta T - \frac{I_Q}{K_C} \tag{5}$$

where K_C is the equivalent thermal conductance of the contacts, given by $K_C = K_{hot}K_{cold}/(K_{hot} + K_{cold})$. Due to the conductive heat current, a change on the electrical current I thus has consequences on the temperature difference $\Delta T'$ and hence on the electromotive force $\alpha\Delta T'$. It is then possible to obtain a genuine Thévenin model of this thermoelectric generator distinguishing constant terms from terms depending on the electrical current I. The voltage output thus reads

$$V = \frac{K_C}{K_0 + K_C}\alpha\Delta T - \left(R + \frac{\alpha^2\overline{T}}{K_0 + K_C}\right)I \tag{6}$$

where the first term of the right hand side is the actual electromotive force [27]. The additional resistive term stems from the introduction of the non-ideal thermal contacts.

2.2. Small-signal electrical model

To perform a potentiostatic impedance spectroscopy measurement, a small ac voltage with sweeping frequency is applied using a lock-in to a circuit made of a sense resistor and of the thermoelectric generator (see, e.g., Ref. [20]); the resulting ac current in the circuit δI and the voltage δV are both measured and then used to compute the complex impedance $\mathscr{Z}$ of the circuit:

$$\mathscr{Z} = -\frac{\delta V}{\delta I}. \tag{7}$$

Note that galvanostatic impedance spectroscopy, performed by