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Global comparison of the goodness-of-fit of wind speed distributions

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ABSTRACT

The aim of this study was evaluating the goodness-of-fit of 24 one-component probability density functions and 21 mixture probability density functions to empirical wind speed probability density functions on a global scale. Era-Interim reanalysis wind speed data for the period 2011-01-01 to 2015-12-31 with a spatial resolution of $1^\circ \times 1^\circ$ were used to compare the goodness-of-fit of 69 combinations of probability density functions and mixture probability density functions fitted with four different parameter estimation methods. The distribution parameters were obtained by applying the moment method, the L-moment method, the maximum likelihood estimation method and the least-squares estimation method. Four goodness-of-fit metrics related to the probability-probability plot, three goodness-of-fit metrics related to the quantile-quantile plot and one goodness-of-fit metric related to the average wind power density were calculated to assess the suitability of distributions. One important result of this study is that mixture probability density functions like the seven-parameter Burr-Generalized Extreme Value, the seven-parameter Dagum-Generalized Extreme Value, the six-parameter Dagum-Weibull and the six-parameter Generalized Extreme Value-Weibull generally provide a superior fit to one-component probability density functions according to goodness-of-fit metrics related to the probability-probability plot. Another important result is that based on the evaluation of goodness-of-fit metrics related to the quantile-quantile plot, the five-parameter Wakeby probability density function is a suitable choice for onshore and the four-parameter Kappa probability density function for offshore wind speed regimes. The four-parameter Johnson system of distributions and Wakeby probability density functions provided the overall best fit for average wind power density. Only for few wind speed regimes, the often used two-parameter Weibull probability density function was identified as the most appropriate distribution. Maps were produced that country-by-country show the most appropriate on- and offshore distributions on a global scale.

1. Introduction

Wind energy is a major renewable energy resource that could supply more than 40 times the annual global electricity consumption [1]. The utilization of wind energy helps to satisfy the rapidly growing energy demand [2]. It also avoids negative effects related to the use of conventional fuels such as greenhouse gas emissions [3]. Moreover human health is protected due to lower emissions of air pollutants [4]. Risks associated with nuclear power plants can be circumvented by the utilization of wind energy [5].

Wind turbines convert the kinetic energy contained in the airflow into electrical energy. The wind turbine power output (P_W) can be defined by the probability density function (pdf) of wind speed (x). The average wind turbine power output ($\overline{P_W}$) is determined by

$$\overline{P_W} = \int_0^\infty P_w(x) f(x) dx \tag{1}$$

where $P_w(x)$ is the wind turbine power output related to x and $f(x)$ is the pdf of x. Therefore, an accurate estimation of $f(x)$ is crucial for the reduction of uncertainty in $\overline{P_W}$ estimation [6].

In the past, the two-parameter Weibull pdf (W) was often used to fit empirical probability density functions (epdfs) computed from measured wind speed data [7]. For instance, Bilir et al. [8] evaluated the wind energy potential in the Incek region in Turkey using W. In another study, the wind energy characteristics in Hong Kong were characterized by applying W [9]. However, results from previous studies demonstrated that W was often not able to reproduce important characteristics of on- and offshore wind speed regimes. Among these are bimodal wind speed regimes [10]. Jung and Schindler [11] found that many other pdfs are better options

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for fitting epdfs that occur in complex terrain than W. According to Carta et al. [12], W offers only a limited fitting accuracy when applying it to wind speed regimes with a large proportion of zero values of x.

To overcome the limited abilities of W to fit epdfs, researchers proposed a large number of one-component pdfs. Akgül et al. [13] found that the two-parameter Inverse Weibull pdf (IW) is a good alternative to W. The two-parameter Nakagami pdf (Na) was satisfactorily applied to epdfs in Iran [14]. In the same region, the two-parameter Lognormal pdf (LogN) was also a proper choice [15]. The application of the three-parameter Burr pdf (B) was found to be successful for describing wind speed characteristics in Southern Italy [16]. Soukissian [17] proposed the four-parameter Johnson SB pdf (JSB) based on its fitting accuracy to wind speed measured at 19 buoys in the Mediterranean Sea. The four-parameter Kappa pdf (K) was found to be suitable for reproducing epdfs in the United Arabian Emirates [18]. According to Jung [19], the annual wind energy yield in areas with mosaic-like land cover pattern and complex topography can be best estimated by the five-parameter Wakeby pdf (Wak).

Beside the one-component pdfs, various mixture pdfs (mpdfs) recently received attention, because of their capacity to reproduce bimodal wind speed regimes and to provide good fit to unimodal wind speed regimes [20]. Shin et al. 2016 [21] suggested the

Table 1
One-component probability density function (pdf), related symbol, number of pdf parameters (N) and pdf equations. Distributions are sorted in ascending order by N.

Pdf	Symbol	N	Equation
Rayleigh	R	1	$f_R(x;\alpha) = \frac{x}{\alpha^2}\exp\left[-\frac{1}{2}\left(\frac{x}{\alpha}\right)^2\right]$
Gamma	G	2	$f_G(x;\alpha,k) = \frac{\alpha^k}{\Gamma(k)}x^{k-1}\exp(-\alpha x)$
Gumbel	Gu	2	$f_{Gu}(x;\alpha,\mu) = \frac{1}{\alpha}\exp\left[-\frac{x-\mu}{\alpha} - \exp\left(\frac{x-\mu}{\alpha}\right)\right]$
Inverse Gaussian	IG	2	$f_{IG}(x;\alpha,\mu) = \sqrt{\frac{\alpha}{2\pi x^3}}\exp\left[-\frac{\alpha(x-\mu)^2}{2\mu^2 x}\right]$
Inverse Weibull	IW	2	$f_{IW}(x;a,k) = \frac{k}{\alpha}\left(\frac{\alpha}{x}\right)^{k+1}\exp\left[-\left(\frac{\alpha}{x}\right)^k\right]$
Logistic	L	2	$f_L(x;\alpha,\mu) = \frac{\exp\left[-\left(\frac{x-\mu}{\alpha}\right)\right]}{\alpha\left\{1+\exp\left[-\left(\frac{x-\mu}{\alpha}\right)\right]\right\}^2}$
Log-Logistic	LogL	2	$f_{\mathrm{LogL}}(x;\alpha,k) = \frac{k}{\alpha}\left(\frac{x}{\alpha}\right)^{k-1}\left[1+\left(\frac{x}{\alpha}\right)^k\right]^{-2}$
Lognormal	LogN	2	$f_{\mathrm{LogN}}(x;\alpha,\mu) = \frac{1}{x\alpha\sqrt{2\pi}}\exp\left\{-\frac{[\ln(x)-\mu]^2}{2\alpha^2}\right\}$
Nakagami	Na	2	$f_{Na}(x;\alpha,k) = \frac{2k^k}{\Gamma(k)\alpha^k}x^{2k-1}\exp\left(-\frac{k}{\alpha}x^2\right)$
Normal	No	2	$f_{No}(x;\alpha,\mu) = \frac{\exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\alpha}\right)^2\right]}{\alpha\sqrt{2\pi}}$
Truncated Normal	N	2	$f_N(x;\alpha,\mu) = \frac{1}{I(\mu,\alpha)\alpha\sqrt{2\pi}}\exp\left[-\frac{(x-\mu)^2}{2\alpha^2}\right]$ where $I(\alpha,\mu) = \frac{1}{\alpha\sqrt{2\pi}}\int_0^\infty \exp\left[-\frac{(x-\mu)^2}{2\alpha^2}\right]dx$
Weibull	W	2	$f_W(x;a,k) = \frac{k}{\alpha}\left(\frac{x}{a}\right)^{k-1}\exp\left[-\left(\frac{x}{\alpha}\right)^k\right]$
Burr	B	3	$f_B(x;\alpha,k,h) = \frac{hk\left(\frac{x}{\alpha}\right)^{h-1}}{\alpha\left[1+\left(\frac{x}{\alpha}\right)^h\right]^{k+1}}$
Dagum	D	3	$f_D(x;\alpha,k,h) = \frac{hk\left(\frac{x}{\alpha}\right)^{hk-1}}{\alpha\left[1+\left(\frac{x}{\alpha}\right)^h\right]^{k+1}}$
Generalized Extreme Value	GEV	3	$f_{GEV}(x;\alpha,k,\mu) = \frac{1}{\alpha}\left[1-\frac{k}{\alpha}(x-\mu)\right]^{\frac{1}{k}-1}\exp\left\{-\left[1-\frac{k}{\alpha}(x-\mu)\right]^{\frac{1}{k}}\right\}$
Log-Pearson 3	LogP	3	$f_{\mathrm{LogP}}(x;\alpha,k,\mu) = [\ln(x)-\mu]^{\beta-1}\frac{\exp\left[-\frac{\ln(x)-\mu}{\alpha}\right]}{\alpha^\beta\Gamma(\beta)}$
Weibull	W3	3	$f_{W3}(x;a,k,\mu) = \frac{k}{\alpha}\left(\frac{x-\mu}{a}\right)^{k-1}\exp\left[-\left(\frac{x-\mu}{\alpha}\right)^k\right]$
Generalized Gamma	GG	4	$f_{GG}(x;\alpha,k,\mu,h) = \frac{k(x-\mu)^{kh-1}}{\alpha^{kh}\Gamma(h)}\exp\left\{-\left[\frac{(x-\mu)}{\alpha}\right]^k\right\}$
Johnson SL	JSL	4	$f(x;\alpha,k,\mu,h) = \frac{k}{\alpha\sqrt{2\pi}}g'\left(\frac{x-\mu}{\alpha}\right)\exp\left\{-\frac{1}{2}\left[h+kg\left(\frac{x-\mu}{\alpha}\right)\right]^2\right\}$ where
Johnson SU	JSU	4	$g(y) = \begin{cases} \ln(y), & \text{for the SL (lognormal)family} \\ \ln(y+\sqrt{y^2+1}), & \text{for the SU (unbounded)family} \\ \ln[y/(1-y)], & \text{for the SB (bounded)family} \\ y, & \text{for the SN (normal)family} \end{cases}$
Johnson SB	JSB	4	
Johnson SN	JSN	4	
			$g'(y) = \begin{cases} \frac{1}{y}, & \text{for the SL (lognormal)family,} \\ 1/\sqrt{y^2+1}, & \text{for the SU (unbounded)family} \\ 1/[y(1-y)], & \text{for the SB (bounded)family} \\ 1, & \text{for the SN (normal)family} \end{cases}$
Kappa	K	4	$f_K(x;\alpha,k,\mu,h) = \alpha^{-1}\left[1-\frac{k(x-\mu)}{\alpha}\right]^{\frac{1}{k-1}}[F(x)]^{1-h}$ where $F_K(x;\alpha,k,\mu,h) = \left\{1-h\left[1-\frac{k(x-\mu)}{\alpha}\right]^{\frac{1}{k}}\right\}^{\frac{1}{h}}$
Wakeby	Wak	5	$f_{WAK}(x;\alpha,\gamma,k,h,\mu) = \{\alpha[1-F(x)]^{\gamma-1}+k[1-F(x)]^{-h-1}\}^{-1}$ where $F^{-1}_{WAK}(x;\alpha,\gamma,k,h,\mu) = \mu+\frac{\alpha}{\gamma}[1-(1-F)^\gamma]-\frac{k}{h}[1-(1-F)^{-h}]$

α: scale parameter.
μ: location parameter.
k: shape parameter.
γ: second scale parameter.
h: second shape parameter.
$\Gamma()$: gamma function.
$F()$: cumulative distribution function.