Engineering Fracture Mechanics 177 (2017) 104–122

Contents lists available at ScienceDirect

Engineering Fracture Mechanics

journal homepage: www.elsevier.com/locate/engfracmech

Multiple solutions in cohesive zone models of fracture

Wu Xu [a,*], Anthony M. Waas [b,*]

[a] School of Aeronautics and Astronautics, Shanghai Jiao Tong University, Shanghai 200240, China
[b] Department of Aeronautics and Astronautics, Guggenheim Hall, 211E, University of Washington, Seattle, WA 98195-2400, United States

ARTICLE INFO

Article history:
Received 16 January 2017
Received in revised form 18 March 2017
Accepted 19 March 2017
Available online 31 March 2017

Keywords:
Cohesive zone model
Cohesive zone size
Fracture mechanics
Non-unique solution

ABSTRACT

The use of cohesive zone models (CZM) for studying crack initiation and propagation can lead to non-unique deformed configurations. This situation can lead to solution branches that may be non-physical, leading to difficulty in interpreting computed results. These aspects are studied in this paper using the double cantilever beam (DCB) specimen first and next in the context of a mode I center crack in a thin sheet of infinite extent subjected to remote tensile loading. Analytical solutions to the CZM that employ constant stress and linear softening laws are presented, and it is shown that when the linear softening cohesive law is used, two cohesive zone sizes are valid for the same external loads, leading to two equally possible deformed configurations, satisfying all the field equations and boundary conditions. The smaller of the two cohesive zones has a lower energy which proposed as a criterion to render a unique solution.

1. Introduction

Strength is an important mechanical property of materials and structures. Fracture mechanics based strength prediction models are widely used in predicting strength of engineering structures. For example, linear elastic fracture mechanics (LEFM) based stress intensity factor (SIF) approach has found much success in the damage tolerance analysis of modern aircraft structure [1–4]. The success of the LEFM approach was somewhat hampered when extended to ductile materials and advanced composite materials. The SIF is valid for small scale damage, where the damage zone size (plastic zone size in metal) at the crack tip is far less than the characteristic length of the cracked specimen. For ductile material and advanced composite materials, the damage zone size may be comparable with the characteristic length. In these cases, the Dugdale-Barenblatt cohesive zone model [5,6] is more versatile, compared to the use of SIF. It can be used for both small and large scale damage [7]. For small scale damage, the cohesive zone model is equivalent to the LEFM, [8,9]. The constant stress cohesive law used in the Dugdale model is a simpler approximation than the nonlinear law used in the original Barenblatt model, even though the crack tip stresses must decrease with distance away from the crack tip. Analytical methods have been frequently used to study the Dugdale strip yield model for fracture, [5,10–12] and fatigue analysis, [13–15] of metal structures.

With the development of advanced numerical methods and implementation using high performance computers, the Barenblatt like cohesive zone models were implemented in the finite element method [6,16–20] for studying crack growth in composite materials, [18,21,20,22,23] and concrete, [24,25], where the assumption of small scale damage zone size may be invalid. Two typical cohesive zone model based finite element methods have been frequently used. One is the cohesive elements implemented in most of the commercial software, for example ABAQUS®. This type of element is used for cases

* Corresponding authors.
E-mail addresses: xuwu@sjtu.edu.cn (W. Xu), awaas@aa.washington.edu (A.M. Waas).

http://dx.doi.org/10.1016/j.engfracmech.2017.03.026
0013-7944/

where the potential crack growth path is known. Cohesive elements are embedded in the potential crack growth path, [17,26–28,20,19]. When the path of cracking is not known a priori, the continuum model and effects of cracking are combined to arrive at a new constitutive law for the element that has a characteristic length scale embedded in the formulation, for example, the crack band model, [29], and the smeared crack approach, [30–33]. If correctly implemented, the predicted strength of a structure should not be dependent on the mesh size [34]. For simple crack configurations, analytical solutions can be obtained. For example, closed-form expressions between the cohesive zone size and the material properties and crack geometries was established by Williams and Hadavinia [35] for the DCB specimen, where, multiple cohesive zone sizes were also found [35]. More extensive solutions to the cohesive zone size for the DCB, end notched flexure (ENF) and mixed-mode bending (MMB) were reported by Xie et al. [36,37], recently. These analytical solutions are useful for guiding the choice of element size in the finite element analysis of cohesive crack initiation and grwoth, [38,39].

Analytical solutions to the cohesive zone models for the DCB and a center crack infinite sheet subjected to remote tensile loading are presented in this paper. The constant stress and linear softening laws are studied. Non-unique cohesive zone sizes for a given load are obtained analytically for both crack configurations. However, the present non-unique solutions are different than what was reported by Williams and Hadavinia [35]. In addition, extensive comparisons of the cohesive DCB for load and displacement control cases are given.

2. Solution to the cohesive DCB

Analytical solutions to the cohesive zone models for five different cohesive laws were given by Williams and Hadavinia [35] for the DCB. The solution was based on using Euler–Bernoulli beam theory for the DCB arms. The cohesive DCB is shown in Fig. 1, where the crack length and beam thickness are represented by a and h, respectively.

The governing equation for the cohesive beam is

$$\begin{cases} \frac{d^4w}{dx^4} = 0, \; -a \leqslant x \leqslant 0 & \text{(a)} \\ \frac{d^4v}{dx^4} = \frac{\sigma}{EI}, \; 0 \leqslant x \leqslant r & \text{(b)} \end{cases} \tag{1}$$

where $w, \; v$ are the opening displacement of the beam, σ is the cohesive stress. For different cohesive laws, the relation between σ and v are different. In this section, the Dugdale constant stress and linear softening law shown in Fig. 1b and c, will be revisited. For the Dugdale constant stress law,

$$\begin{cases} \sigma = -b\sigma_u & \text{(a)} \\ G_c = \sigma_u v_0 & \text{(b)} \end{cases} \tag{2}$$

for the linear softening law,

$$\begin{cases} \sigma = -b\frac{\sigma_u}{v_0}(v_0 - v) & \text{(a)} \\ G_c = \frac{1}{2}\sigma_u v_0 & \text{(b)} \end{cases} \tag{3}$$

where b is the width of the beam, σ_u is cohesive tensile strength. Here, G_c is half of the cohesive fracture energy G_{IC}, due to only one beam being used in the following calculation of the crack opening v. It is observed that the critical crack opening v_0 for the linear softening law is two times of that in the Dugdale constant stress law. The solutions to these two different cohesive laws given in [35] will be reproduced as follows.

2.1. Solution to constant stress law

Substituting the cohesive law Eq. (2) into Eq. (1b), the governing equation for the beam, $0 \leqslant x \leqslant r$ is given as follows.

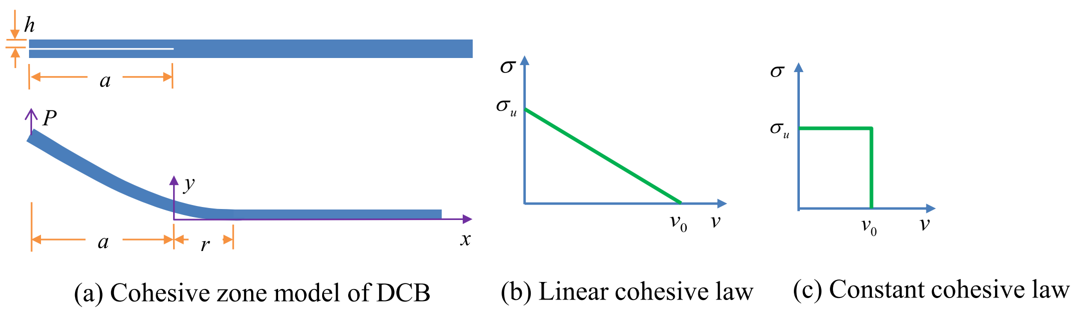

Fig. 1. Double cantilever beam with different cohesive laws.