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Energetical formulation of size effect law for quasi-brittle fracture

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ABSTRACT

A criterion for quasi-brittle failure based on local quantities is proposed. The propagation of long cracks is controlled by the energy release rate, while the initiation of the crack is shown to be well described by the derivative of the energy release rate with respect to the crack length. An equation based on well-known models for the analysis of the size and boundary effects is then adapted to describe the competition between these two quantities in order to characterize the local failure. The predictions of rupture of pre-cracked specimens of different sizes and materials are compared to the experimental results given in the literature. The generality of the formulation for describing the crack propagation mechanisms for other defects, such as v-notches and holes, is finally discussed.

1. Introduction

The stress and energy release are basic elements found in most rupture criteria where the failure is characterized by the initiation and propagation of cracks. For an intact structure, a simple stress criterion may indicate crack initiation for a given stress state that is defined as the material strength. On the other hand, considering an energy criterion, a large crack may propagate if the energy release rate during its propagation reaches a certain value associated with the fracture energy of the material. Intermediate situations, such as the rupture of a small cracked specimen or structures presenting "imperfections" other than cracks (e.g. heterogeneities, complex shaped notches, etc.) are not well described by either of these two criteria. These difficulties have given rise to different approaches in order to predict the rupture behavior of quasi-brittle materials.

Based on the stress criteria, Critical Distance Theories (CDTs) were initially proposed in the 1930s by Neuber [1] and Peterson [2] using a punctual evaluation of the stress at a certain distance from the crack tip (Point Stress Method) and the average of the stress over a certain length (Average Stress Method), respectively, in order to analyze the fatigue failure of metallic structures. Adopting the same concept, Novozhilov [3] proposed a simple failure criterion based on the average normal stress along the anticipated path of the crack formation. This model has been expanded by Seweryn [4–7] to study both regular and singular stress concentrations under mode I or mixed mode loading.

Leguillon [8] proposed a criterion for failure initiation at a sharp v-notch under mode I loading, which requires the stress condition and energy condition to be fulfilled simultaneously. This criterion was compared with several known failure initiation criteria and validated in [8,9] for mode I loading. It was improved by Leguillon and Yosibash [10] by introducing a correction due to the small notch tip radius, later extended by Yosibash et al. [11] to mixed mode loading and validated

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Nomenclature

a, a_t	crack length and transition crack length
a_f, a_{vr}	propagation length scales
A_i	geometrical correction factor
B	positive dimensionless factor in the size effect law
c	v-notch depth
dG/da, $max\ dG/da$	derivative of the energy release rate and its critical value
D	diameter of the circular hole
E	Young's modulus
f_t	material tensile strength
F_0, F_N, F_{max}	applied load, failure load, and maximum failure load
G_0, G_c	energy release rate and fracture energy
h	height of the beam
K_0, K_c	stress intensity factor and fracture toughness
K_0^N, K_c^N	generalized stress intensity factor and its critical value
l_{ch}	Irwin's characteristic length
L	span of the beam
r	positive dimensionless factor in the proposed model
$\bar{r}$	positive dimensionless factor in the size effect law
R	radius of the circular hole
t	thickness of the specimen
w	width of the plate
W, W_0	size and transition size of the structure
γ	notch opening angle
η	dimensionless parameter for the energy release rate of a short crack emanating from the v-notch tip
λ	order of stress singularity
Λ	dimensionless parameter for the stress intensity factor of a short crack emanating from the v-notch tip
ν	Poisson ratio
ξ	ratio of K_0^N and F_0
$\sigma_0, \sigma_N, \sigma_{max}$	applied stress, nominal strength, and maximum failure stress
$\sigma_N^{strength}$	nominal strength predicted by the strength mechanism

by experimental observations. The concept of Finite Fracture Mechanics was used in Leguillon's criterion, which assumes the instantaneous formation of cracks of finite size at initiation [12]. Instead of the point-wise stress criterion adopted in Leguillon's criterion, Cornetti et al. [13] proposed a similar criterion, based on the evaluation of stresses prior to fracture averaged over the crack. These coupled criterion allows for the general analysis of arbitrary stress concentrations [14–16], and has been used by many researchers to establish general failure criteria for a wide range of engineering problems in the last 14 years [12].

The cohesive zone models [17–19] simulate the damage that occurs in the process zone located ahead of the crack tip. This approach, which involves nonlinear constitutive laws that are described by a displacement jump and corresponding traction along the interfaces, provides a phenomenological model with which to simulate complex fracture behavior, such as crack nucleation, initiation and propagation [20]. The extension of the classical cohesive model to quasi-brittle materials usually shows fractal patterns in the failure process. This fractal approach leads to a scale-invariant cohesive crack model which is able to predict the size effects even in tests where the classical approach fails, e.g. the direct tension test [21].

Most of the methods are associated with a length scale that is usually proportional to Irwin's [22] characteristic length l_{ch}, which depends on the properties of the material, such as the stiffness, strength and fracture toughness. However, for certain building materials (such as concrete, rocks, and some types of ceramics), the value of the characteristic length scale can become too large when compared to the specimen size, and sometimes can even exceed it, which makes the direct implementation of these approaches impossible [23]. In the discrete element methods, the materials are organized into assemblies of particles in contact. Initially developed by Cundall and Strack [24] for modelling granular and particulate systems, these methods were further adapted to study the fracture of quasi-brittle materials, such as concrete [25,26] and rocks [27]. Despite the simple (and physical) local point of view of ruptures, the intrinsic scale effects related to the particle size and characteristic length l_{ch}) also affect the response of the model to quasi-brittle rupture [28–30].

This paper presents an alternative description of quasi-brittle failure based only on local quantities that are related to the energy release rate G. Hence, the size [31,32] and boundary effects [33–35] that are observed in the experiments are simply analyzed through the combination of mechanisms related to the strength and fracture toughness of the material, thereby avoiding some of the nonphysical aspects associated with most of the existing models. In Section 2, the notions of the size and boundary effects are presented. Based on the existing expressions used to describe these types of experimental evidence,