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# On the calculation of plastic strain by simple method under non-associated flow rule

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A R T I C L E I N F O

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The non-associated flow rule (non-AFR), which has been proved with advantages in accurately predicting yield stress components and Lankford coefficients, has different definitions for yield function and plastic potential function. Currently, there are two typical approaches to determine the plastic strain for non-AFR, one is called simple method which still uses the equivalent plastic strain (EPS) to calculate the plastic strain tensor, while the other is called full method which adopts the plastic compliance factor. Many researchers prefer to adopt the simple method in calculating the plastic strain due to its higher computation efficiency and its acceptable accuracy like the full method for some materials. However, the limitation and application condition of the simple method are still ambiguous. In the present work, a restriction for calculating the parameters in plastic potential function for the simple method is clarified and the limitation of the simple method has also been investigated. It is found that if the relative error of the difference between the yield function value and that of plastic potential function is within ±5% and hardening exponent is in the range of 0.1–0.3, the maximum absolute value of the relative error of the stress predicted by the simple method can be controlled less than 1.6%. Besides, an effective approach is introduced to reduce the error of the stress predicted by these two methods, which can improve the applicability of the simple method for those materials with the relative error of the difference between the value of yield function and that of plastic potential function is less than 10%. In order to have deeper understanding about their applicability, the two methods are compared under uniaxial tension along different orientations for AA2090-T3 and AA2008-T4 with the fully implicit return-mapping scheme. Fracture simulations of AA2090-T3 have also been conducted by introducing different methods. The results show that during the loading in transverse direction, the difference of EPSs predicted by these two methods is very small for AA2008-T4, while for AA2090-T3, the EPS predicted by the simple method is almost twice of that calculated by the full method. This further indicates that the applicability of the simple method mainly depends on the characteristics of materials.



## 1. Introduction

The sheet metal materials during cold forming demonstrates obvious anisotropy as the result of crystallographic texture after cold or hot rolling. This characteristic plays an important role in the distribution of strain and stress, which finally influences the residual stress and shape of the metallic parts. The study of anisotropic yield functions based upon the Associated Flow Rule (AFR) hypothesis has developed and been widely used. Under this hypothesis, both the yielding and plastic flow are determined by the same anisotropic yield function.

To describe the behavior of sheet metal, various yield functions have been developed (Hill, 1948; Hosford, 1972; Hu, 2005; Karafillis and Boyce, 1993; Barlat et al., 2003, 2005; Cazacu and Barlat, 2003; Cazacu et al., 2006). Although the accuracy of prediction has been improved, the number of parameters in some yield functions is more than 10, which will cost more computational time in FE simulation. Most of the yield functions described above are just suitable for yielding prediction or anisotropic coefficient prediction for some materials with higher anisotropy such as AA2090-T3 (Park and Chung, 2012). Instead of developing a new yield function with more parameters under AFR framework, another approach to solve this problem is non-AFR, which describe the plastic flow and yielding direction with two independent functions. A

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straightforward and convenient way to define a non-AFR model of non-AFR is using the classical yield function in AFR model to define the yield function and plastic potential function in non-AFR model (Stoughton, 2002; Stoughton and Yoon, 2006; Cvitanic et al., 2008). Stoughton (2002) developed a non-AFR model based upon Hill'48 formulation, which could describe the yield stress and Lankford coefficients along rolling, diagonal and transverse directions. Stoughton and Yoon (2006) clarified that the AFR is not necessary for stability of metal flow, which means the plastic potential function and yield function can be defined separately. The Hill'48 non-AFR and Karafillis–Boyce1993 (KB93) non-AFR were compared with the experimental data by Cvitanic et al. (2008) under the premise of the principle of plastic work equivalence. And a FE formulation for non-AFR based on the research of Yoon et al. (1999) was also provided by them. Taherizadeh et al. (2010) introduced the mixed-hardening model into a non-AFR with the form of Hill'48, which can improve the description of both anisotropy and hardening. Three AFR models and three non-AFR models were compared by Mohr et al. (2010) to predict the characteristic of sheet metal under multi-axial loading condition. A non-AFR model with the symmetric stiffness modulus by combining isotropic-kinematic hardening law was successfully utilized by Park and Chung (2012) to predict the ears of AA5042 and AA2090-T3 sheets during cup drawing. The accuracy of Yld 2000-2d non-AFR for predicting cup height and ears of AA2090-T3 sheets have also been verified by Safaei et al. (2013). Paulino and Yoon (2015) replaced the yield function of Yld 2000-2d non-AFR with Hill'48 yield function to simulate mini-die cup drawing of 5019A-H48, whose accuracy of earing prediction is the same as Yld 2004-18p AFR model but the computational time is half of Yld 2004-18p AFR model. Safaei et al. (2014) also compared the simple method and the full method of Yld 2000-2d non-AFR by FE simulation of cylindrical cup deep drawing of AA2090-T3. It is found that the computational time for the simple method is 66.17 min, while for the full method is 82.65 min, which shows that the computational time has been improved about 20% if the simple method is used. The non-AFR theory has also been utilized to improve the prediction of damage, cyclic response and springback of sheet metals under different loading conditions (Gao et al., 2011; Roth and Mohr, 2016; Oya et al., 2014; Ghaei and Taherizadeh, 2015; Tancogne-Dejean et al., 2016).

More efforts have been made on non-AFR models as a more convenient way to describe yield stresses and anisotropic coefficients. In order to implement these non-AFR models described above into FE software, all kinds of integration schemes have been derived by combining isotropic hardening, isotropic-kinematic hardening or mixed-hardening (Cvitanic et al., 2008; Taherizadeh et al., 2011; Park and Chung, 2012; Safaei et al., 2015; Wali et al., 2016). However, it should be noted that most non-AFR models are coupled with the equivalent plastic strain (EPS) during derivation, while the others adopt plastic compliance factor. Mohr et al. (2010) found that these two methods show little difference in predicting stress-strain curve for DP590 sheet metal with isotropic hardening under multi-axial loading. Safaei et al. (2014) also compared the simple method with the full method and found that the simple method would overestimate the EPS or stress. Then they proposed a scaled simple method, which has the same accuracy of the full method to predict the earing but less computational time. However, they ignored the restriction for calculating the parameters in plastic potential function of non-AFR with the simple method. Besides, if the complex mixed hardening model is introduced in non-AFR, the numerical description for the full method would be very laborious due to the complex linearization and stress-update algorithm. Hence, it is necessary to investigate the application conditions of the simple method. Here, we will make a comprehensive comparison between the simple method and the full method and discuss the scope of application of the simple method.

## 2. Review of non-AFR

In the non-AFR model, the yield function $f_y$ determines the yielding and the plastic potential function $f_p$ determines the rate and direction of plastic strain. The parameters in yield function and plastic potential function are determined respectively by the yield stresses and Lankford coefficients along different orientations under unidirectional loading and balanced biaxial tension. The increment of plastic strain tensor in non-AFR can be expressed as follow

$$d\boldsymbol{\varepsilon}^{\mathbf{p}} = d\lambda \frac{\partial f_p}{\partial \boldsymbol{\sigma}} \tag{1}$$

where $d\boldsymbol{\varepsilon}^{\mathbf{p}}$ is the plastic strain increment, $d\lambda$ is the plastic compliance factor, $\boldsymbol{\sigma}$ is the Cauchy stress tensor.

Two symbols $\mathbf{G}$ and $\mathbf{P}$ are introduced for conveniently describing the direction of yielding and plastic flow.

$$\mathbf{G} = \frac{\partial f_y}{\partial \boldsymbol{\sigma}}, \ \mathbf{P} = \frac{\partial f_p}{\partial \boldsymbol{\sigma}} \tag{2}$$

### 2.1. The full method and the simple method

The full method obeys the principle of plastic work equivalence.

$$f_y d\bar{\varepsilon}_p = \boldsymbol{\sigma} : d\boldsymbol{\varepsilon}^{\mathbf{p}} \tag{3}$$

where $d\bar{\varepsilon}_p$ is EPS increment.

According to Euler's identity, for the first order homogeneous function, we can get Eq. (4)

$$f_p = \boldsymbol{\sigma} : \frac{\partial f_p}{\partial \boldsymbol{\sigma}} \tag{4}$$

Substituting Eq. (1) and Eq. (4) into Eq. (3)

$$d\lambda = d\bar{\varepsilon}_p \frac{f_y}{f_p} \tag{5}$$

If Eq. (5) is used in non-AFR, the method is called the full method. If Eq. (6) is adopted, the method would be the simple method.

$$d\lambda = d\bar{\varepsilon}_p \tag{6}$$

In the next sections, we will mainly discuss the stress and strain condition under uniaxial loading for both methods. For the full method, the Eq. (3) can be transform to Eq. (7).

$$d\bar{\varepsilon}_p^{full} = \frac{\sigma_u}{f_y} d\varepsilon_{u\theta}^p \tag{7}$$

where $d\bar{\varepsilon}_p^{full}$ is EPS increment calculated by the full method, $\sigma_u$ and $d\varepsilon_{u\theta}^p$ are the yield stress and plastic strain increment along uniaxial loading, respectively.

For the simple method, we substitute Eq. (6) into Eq. (1) and multiply both sides by $\sigma_u$

$$\sigma_u d\varepsilon_{u\theta}^p = \sigma_u d\bar{\varepsilon}_p^{simple} \frac{\partial f_p}{\partial \sigma_u} = d\bar{\varepsilon}_p^{simple} f_p \tag{8}$$

where $d\bar{\varepsilon}_p^{simple}$ is EPS increment calculated by the simple method.