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Higher order interfacial effects for elastic waves in one dimensional phononic crystals via the Lagrange-Hamilton's principle

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ABSTRACT

This work proposes new transmission conditions at the interfaces between the layers of a three-dimensional composite structures. The proposed transmission conditions are obtained by applying the asymptotic expansion technique in the framework of Lagrange-Hamilton's principle. The proposed conditions take into account interfacial effects of higher order, thus representing an extension of the classical zero-thickness interface models. In particular, the (small) thickness of the interface together with its inertia, stiffness and anisotropy are accounted for. The effect of the transmission conditions on the band structure of Bloch–Floquet waves propagating in a one dimensional phononic crystal is discussed based on numerical results.

1. Introduction

Phononic crystals are composites with a periodic structure made of materials with different elastic constants and densities. In these composites, elastic waves with frequencies within a specific range (the phononic bandgap) are not allowed to propagate. Therefore, phononic crystals present innovative filtering properties and offer possibilities for controlling sound and heat propagation (Kushwaha et al., 1993; Jensen, 2003; Ghazaryan and Piliposyan, 2011; Maldovan, 2013).

Many authors have studied the effect of material properties on phononic band gaps (see (Vasseur et al., 2001; Wu et al., 2004; Maldovan, 2013; Chen et al., 2014) and references therein) and found that microstructured materials are able to control sound, whereas to control heat, nanostructures are generally required. For a fine-scaled material with a large ratio of interfacial region to the bulk, the influence of surface characteristics can be substantial and it is thus fundamental to propose reliable and efficient models able to account for interfacial effects.

In the literature, a very large number of interface model have been developed (see, for example (Challamel and Girhammar, 2011; Benveniste and Miloh, 2001; Hashin, 2002; Klarbring, 1991; Bövik, 1994; Lebon and Zaittouni, 2010; Nairn, 2007; Benveniste, 2013; Li et al., 2015)). We can classify these models into two large families: phenomenological models, essentially built from experimental data, and deductive models based on micro-mechanical analyses. In the present paper, we deal with the second family.

The application of asymptotic techniques to obtain models of interfaces is now well established (Benveniste, 2006; Krasuki and Lenci, 2000; Lebon et al., 1997; Rizzoni and Lebon, 2012, 2013; Rizzoni et al., 2014; Serpilli, 2015; Serpilli and Lenci, 2016). The idea behind this application is the replacement of a thin, elastic, anisotropic interphase by a proper interface model; the equivalence between the two models is established by studying the asymptotic behavior of the interphase as its thickness becomes smaller and smaller. In Section 2, the problem of a composite made of three deformable solids (two adherents and a thin interphase) perfectly bonded together is introduced in the framework of elastodynamics. The Lagrangian problem is introduced and expanded with respect to the small parameter (the interphase thickness). In Section 3, four sub-problems are studied, allowing us to derive the interfacial displacement and traction jump relations at each level of expansion. In particular, higher order levels of the expansion are taken into account.

In Section 4, the jump relations are reformulated into a general elastic imperfect interface model in such a way that they take

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simpler and compact but equivalent forms, particularly convenient for later use. It is also shown that the present formulation unifies and extends four widely used interface models, the perfect interface, the mass interface, the spring interface and the spring-mass interface, thus representing an enrichment of the classical interface models due to high order interfacial effects. Notably, the presence of first order derivates of the displacement and stress vector fields make the proposed imperfect interface model nonlocal in character.

In Section 5, the imperfect interface model is applied to estimate the interfacial effects of the periodic structure on bandgaps of a one-dimensional phononic crystal with imperfect contacts between the two constituent layers. The standard transfer matrix approach is employed (Lekner, 1994; Rokhlin and Wana, 1991; Rokhlin and Wang, 1992). In particular, an additional interlayer matrix is introduced, taking into account the imperfect contact. The dispersion equation is solved numerically and the dispersion curves are shown in the Brillouin zone. The band gaps of the phononic crystal with imperfect contact are compared with those obtained with perfect contact. In particular, the effects of the small thickness of the imperfect interface, of its inertia and stiffness on the band structure of the laminated phononic crystal are discussed on the basis of the numerical results.

2. Statement of the problem

In the following a composite body made of three deformable solids, two elastic adherents and a thin elastic adhesive, is considered (cf. Fig. 1). At the initial time t_1, the composite occupies the bounded domain Ω^ε depending on a small parameter ε which is the constant thickness of the adhesive. An orthonormal Cartesian basis $(O, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ is introduced and $\mathbf{x} = (x_1, x_2, x_3)$ is taken to denote the position of a particle. The adhesive occupies the initial domain B^ε, defined by $B^\varepsilon = \left\{ \left(x_1, x_2, x_3\right) \in \Omega^\varepsilon : \left|x_3\right| < \frac{\varepsilon}{2} \right\}$. We take ∂B^ε to denote the boundary of B^ε, which is supposed to be sufficiently smooth. Thus, the origin of the Cartesian basis lies at the center of the adhesive midplane and the x_3- axis runs perpendicular to the open bounded set $S = \{(x_1, x_2, x_3) \in \Omega^\varepsilon : x_3 = 0\}$, which in the following will be called the interface. The adherents occupy respectively the initial domains $\Omega^\varepsilon_\pm$ defined by $\Omega^\varepsilon_\pm = \left\{ \left(x_1, x_2, x_3\right) \in \Omega : \pm x_3 > \frac{\varepsilon}{2} \right\}$. We take $\partial\Omega^\varepsilon_\pm$ to denote the boundary of $\Omega^\varepsilon_\pm$, which is supposed to be sufficiently smooth. The two-dimensional domains $S^\varepsilon_\pm$ are taken to denote the interfaces between the adhesive and the adherents, $S^\varepsilon_\pm = \left\{ \left(x_1, x_2, x_3\right) \in \Omega : x_3 = \pm\frac{\varepsilon}{2} \right\}$. On a part $S^\pm_g$ of the boundary

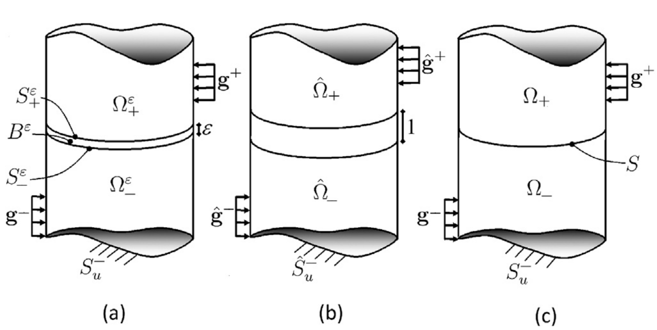

Fig. 1. Geometry of the composite. Initial reference configuration made of two adherents in perfect contact with a thin adhesive (a); corresponding rescaled configuration with an adhesive of unit thickness (b); limit configuration obtained as the thickness ε of the adhesive goes to zero (c).

$\partial\Omega^\varepsilon/S^\varepsilon_\pm$, an external time-dependent load $\mathbf{g}^\pm(t, \mathbf{x}), t \in (t_1, t_2)$, is applied, and on a part $S^\pm_u$ of $\partial\Omega^\varepsilon/S^\varepsilon_\pm$ such that $S^\pm_g \cap S^\pm_u = \emptyset$, the displacement is imposed to vanish. Moreover, it is assumed that $S^\pm_u \cap B^\varepsilon = \emptyset$, $S^\pm_g \cap B^\varepsilon = \emptyset$ and $S^\pm_u \cup S^\pm_g \cup S^\varepsilon_\pm = \partial\Omega^\varepsilon_\pm$. We take S^ε_l to denote $\partial B^\varepsilon/S^\varepsilon_\pm$. The part of the boundary S^ε_l is force free. A time-dependent body force $\mathbf{f}^\pm(t, \mathbf{x}), t \in (t_1, t_2)$, is applied in $\Omega^\varepsilon_\pm$. Let $\mathbf{g}^\pm$ and $\mathbf{f}^\pm$ be assumed regular functions on $[t_1, t_2] \times S_g$ and $[t_1, t_2] \times \Omega^\varepsilon_\pm$, respectively. In the following, $\mathbf{u}^\varepsilon(t, \mathbf{x})$ is taken to denote the displacement field, $\boldsymbol{\sigma}^\varepsilon(t, \mathbf{x})$ the Cauchy stress tensor and $\mathbf{e}(\mathbf{u}^\varepsilon)$ the strain tensor. Under the small strain hypothesis we have $e_{ij}(\mathbf{u}^\varepsilon) = \frac{1}{2}(u^\varepsilon_{i,j} + u^\varepsilon_{j,i})$, where the comma is the partial derivative.

The two adherents and the adhesive are supposed to be elastic, thus

$$\boldsymbol{\sigma}^\varepsilon = \mathbf{a}_\pm \mathbf{e}(\mathbf{u}^\varepsilon) \quad \text{in } \Omega^\varepsilon_\pm, \tag{1}$$

$$\boldsymbol{\sigma}^\varepsilon = \mathbf{b}^\varepsilon \mathbf{e}(\mathbf{u}^\varepsilon) \quad \text{in } B^\varepsilon. \tag{2}$$

The elasticity tensors $\mathbf{a}_\pm$ and $\mathbf{b}^\varepsilon$ have the usual properties of symmetry, $S_{ijhk} = S_{hkij} = S_{jikh}$, and of positivity, i.e. there exists $\alpha > 0$ such that

$$S_{ijhk} e_{ij} e_{hk} > \alpha e_{ij} e_{ij}, \quad e_{ij} = e_{ji}. \tag{3}$$

We take $\rho^\pm$ and ζ^ε to denote the strictly positive volumetric mass densities in the adherents and in the adhesive, respectively, and $[[]]$ to denote the jump along $S^\varepsilon_\pm$. The equations governing the motion of the composite structure are written as follows:

$$\begin{cases} \sigma^\varepsilon_{ij,j} + f^\pm_i = \rho_\pm \ddot{u}^\varepsilon_i & \text{in } [t_1, t_2] \times \Omega^\varepsilon_\pm, \\ \sigma^\varepsilon_{ij} n_j = g^\pm_i & \text{on } [t_1, t_2] \times S^\pm_g, \\ \sigma^\varepsilon_{ij,j} = \zeta^\varepsilon \ddot{u}^\varepsilon & \text{in } [t_1, t_2] \times B^\varepsilon, \\ [[u^\varepsilon_i]] = 0 & \text{on } [t_1, t_2] \times S^\varepsilon_\pm, \\ u^\varepsilon_i = 0 & \text{on } [t_1, t_2] \times S^\pm_u, \\ \sigma^\varepsilon_{ij} = a^\pm_{ijhk} e_{hk}(u^\varepsilon) & \text{in } [t_1, t_2] \times \Omega^\varepsilon_\pm, \\ \sigma^\varepsilon_{ij} = b^\varepsilon_{ijhk} e_{hk}(u^\varepsilon) & \text{in } [t_1, t_2] \times B^\varepsilon, \\ u^\varepsilon_i = U^\varepsilon_i, & \text{for } t = t_1, \text{ in } \Omega^\varepsilon, \\ \dot{u}^\varepsilon_i = v^\varepsilon_i, & \text{for } t = t_1, \text{ in } \Omega^\varepsilon, \end{cases} \tag{4}$$

where $\dot{u}_i$ and $\ddot{u}_i$ are the first and second derivatives in time of u_i, respectively, and U_i (resp. v_i) are the initial displacement (resp. velocity) data. It is remarked here, that (4) implies that $[[\sigma^\varepsilon_i]] = 0$ on $[t_1, t_2] \times S^\varepsilon_\pm$. Note that b^ε_{ijhk} and ζ^ε can depend on ε. If $f^\pm \in L^2([t_1, t_2]; H^1(\Omega^\varepsilon, R^3))$ and $g^\pm \in L^2([t_1, t_2]; H^1(S^\pm_g, R^3))$, then problem (4) has an unique solution in $H^1([t_1, t_2]; H^1(\Omega^\varepsilon, R^3))$ (Ciarlet, 1976; Lions and Magenes, 1968). In the following, we take $\|\|$ to denote the usual euclidian norm in R^3. The Lagrangian is introduced

$$\mathscr{L}(\mathbf{u}^\varepsilon) = T^\varepsilon(\dot{\mathbf{u}}^\varepsilon) - E^\varepsilon(\mathbf{u}^\varepsilon), \tag{5}$$

where T^ε is the total kinetic energy, sum of the kinetic energies of the adherents and the adhesive,

$$\begin{aligned} T^\varepsilon(\dot{\mathbf{u}}^\varepsilon) &= T^\varepsilon_+(\dot{\mathbf{u}}^\varepsilon) + T^\varepsilon_-(\dot{\mathbf{u}}^\varepsilon) + T^\varepsilon_B(\dot{\mathbf{u}}^\varepsilon), \\ T^\varepsilon_\pm(\dot{\mathbf{u}}^\varepsilon) &= \frac{1}{2} \int_{\Omega^\varepsilon_\pm} \rho_\pm \|\dot{\mathbf{u}}^\varepsilon\|^2 dx, \\ T^\varepsilon_B(\dot{\mathbf{u}}^\varepsilon) &= \frac{1}{2} \int_{B^\varepsilon} \zeta^\varepsilon \|\dot{\mathbf{u}}^\varepsilon\|^2 dx, \end{aligned} \tag{6}$$

and E^ε is the total potential energy