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Effect of thermal gradation on steady state creep of functionally graded rotating disc

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ABSTRACT

The paper presents the steady state creep behavior of isotropic rotating disc made of parabolically varying functionally graded material in the presence of thermal gradient. The creep rates have been obtained for the discs rotating at elevated temperatures. Investigations for disc rotating at uniform temperature from inner to outer radii has been done using von Mises' yield criterion. Further, work has been extended for discs rotating at parabolically decreasing temperature. The results are exhibited graphically for the said temperature profiles. A small variation is observed for radial and tangential stresses for said thermal gradations. However, in the presence of thermal gradation the strain rates vary significantly as compared to disc at uniform temperature. Thus in functionally graded rotating disc the temperature gradation significantly effect the creep behavior of a rotating disc.

1. Introduction

Advancement in technology has made it possible to fuse materials with components that display modification in their properties. The conventional materials like metals or ceramics might not sustain alone under high temperature or thermal gradient. Thus, functionally graded materials (FGMs) were invented and thus led to the development of high-calibre heat resistant materials (Singh and Gupta (2014)). The concept of FGM was proposed in 1984 by material scientists in the Sendai area of Japan. Uemura (2003) suggested many commercial applications of FGM such as machining tool, optical fiber, optical filter, surface material for wristwatch, blade of electric shaver, spike for baseball shoes, etc. The design, process and characteristics of FGM have been described by Hirai and Chen (1999). With the introduction of graded structures, many fabrication processes and various new materials with unique properties have been developed and prepared. The constituents or their contents vary in some direction in FGMs thus enhancing the performance of these materials. FGMs have been designed in ultra high temperature resistant materials for numerous applications in aircrafts, space vehicles, and other components working at elevated temperature. FGMs are used for components which are subjected to high mechanical and thermal loadings because of their unique performance due to spatial tailoring of properties at a microscopic level as in Singh and Gupta (2014). Suryanarayanan et al. (2013) studied the use of Al-SiC metal matrix composite (MMC) which have applications in the aerospace industry. Duc and Thang (2014) investigated the nonlinear static buckling for imperfect eccentrically stiffened functionally graded thin circular cylindrical shells with temperature dependent properties surrounded on elastic foundation in thermal environment. Both shells and stiffeners were deformed simultaneously due to temperature. Material properties were graded in the thickness direction according to Sigmoid power law distribution in terms of volume fractions of constituents (S-FGM) with meta-ceramic-metal layers. Numerical results were given for evaluating effects of function and Bubnov-Galerkin method were applied. Duc and Quan (2015) presented an analytical investigation on the nonlinear dynamic analysis of functionally graded double curved tin shallow shells using a simple power-law distribution (P-FGM) with temperature-dependent properties on n elastic foundation and subjected to mechanical load and temperature. The obtained results showed the effects of temperature, material and geometrical properties, imperfection and elastic foundation on the nonlinear vibration and nonlinear dynamical response of double curved FGM shallow shells.

Gupta et al. (2004a,b) investigated the creep behavior of a rotating disc having thermal gradient in the radial direction made of isotropic FGM by Sherby's law. The analysis indicated that the steady-state strain rates were significantly lower in disc with linear particle distribution as compared to that in an isotropic disc with uniform distribution. It was concluded that the strain rates in discs operating under thermal gradient were less in comparison to similar discs at uniform temperature.

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Further, Gupta et al. (2005) carried out creep behavior of a rotating disc made of FGM having decreasing linear distribution of SiC particles from inner to outer radii in matrix of pure aluminium. The presence of thermal gradation and particle gradient either separately or simultaneously caused lowering of steady state creep rates when compared with the disc having uniform particle distribution and operating under isothermal conditions was observed. Rattan et al. (2010) studied the creep analysis for isotropic axisymmetric rotating disc made of particle-reinforced FGM. The result obtained for disc with non-linear variation of particle distribution along the radial distance were compared with discs having uniform or linear particle distribution along the radial distance. Functionally graded rotating disc with parabolic profile proved to be more superior than disc with uniform or linear distribution of particle. Durodola and Attia (1999) explored the advantages of using rotating hollow and solid disks made up of fibre-reinforced functionally graded materials. Deformation and stress distribution in the disks were predicted by finite element method and numerical integration of the governing differential equations. Mangal et al. (2013) investigated the steady-state creep in a cylinder made up of functionally graded material rotating at uniform angular speed. The thermal gradient in the rotating cylinder of FGM was calculated using finite element method. The study revealed that the presence of particle gradient & thermal gradient significantly affected the radial, axial and tangential and effective stresses in the cylinder. Rattan et al. (2016) investigated the effect of thermal residual stress on steady-state creep behavior of thermally graded isotropic disc rotating at elevated temperature. It was observed that due to the presence of thermal residual stress there is a significant change in the stress distribution. The creep analysis was carried out using isotropic Hoffman yield criterion. Bose and Rattan (2016) made an attempt to model steady-state creep for thermally graded rotating disc made of linearly varying functionally graded material. The creep distributions have been obtained for the discs rotating at linearly and parabolically decreasing temperatures. The analysis indicated that stress in disc operating under thermal gradient slightly increases in comparison to disc operating at constant temperature. Thakur et al. (2016) studied thermal creep stresses and strain rates using Seth's transition theory in a circular disc with shaft having variable density. It was observed that radial stress at the internal surface of the disc made of incompressible material was increased due to thermal effect as compared to tangential stress and also radial stress increases further with the increase in angular speed as compared to without thermal effect. In discs made of compressible material, strain rates have maximum values at the internal surface. Dharmpal et al. (2015) developed mathematical model to investigate steady state creep in a functionally graded rotating disc having variable thickness. The SiCp content was assumed to decrease from the inner to outer radius of the disc. The creep behavior of the disc material was described by threshold stress based law with a stress exponent of 5. The stresses and strain rates have been estimated for similar FGM discs with three different thickness profiles i.e. constant thickness, linearly varying thickness and hyperbolic varying thickness. The FGM disc having hyperbolic thickness profile exhibited lowest stresses and strain rates compared to linear or constant thickness disc. The tangential and radial strain rates in FGM discs with linear and hyperbolic thickness profiles were respectively lowered by about two and three orders of magnitude when compared to constant thickness FGM disc. The FGM discs having linear and hyperbolic thickness profiles possessed lesser chances of distortion due to relatively uniform distribution of radial strain rate. Khanna et al. (2015) investigated secondary creep in a rotating Al-SiCp disc having different thickness profiles and reinforcement (SiCp) gradients. The creep behavior was described by threshold-stress based law and yield following Tresca criterion. It was observed that as the thickness gradient of the disc was increased, at the inner radius the radial stress decreased but at the outer radius it increased, whereas the tangential stress decreased over the entire radius. The radial stress also increased throughout the radial distance as SiCp increased in the FGM disc, however, at the inner radius the tangential stress increased but decreased towards the outer radius. Thus, the composite disc having higher thickness and higher reinforcement gradients exhibited lesser distortion. Duc et al. (2010) presented the bending analysis of a three-phase composite plate with an epoxy matrix, reinforced glass fiber and titanium oxide particles. The results indicated that creep strains lead to compression in the composite material introducing reinforced fibers and particles reduced the plate's deflection while increasing the stretch coefficient. Gupta and Singh (2016) provided an analytical framework for the analysis of creep stresses and creep rates in the isotropic rotating non-FGM/FGM disc with uniform and varying thickness. The creep response of rotating disc is expressed by threshold stress taking 8 as the stress exponent. The results obtained for isotropic non-FGM or FGM disc having constant thickness have been compared to those estimated for isotropic disc having varying thickness with the same average particle content distributed uniformly.

Thus, the present paper aims to study the effect of thermal gradation on steady state creep rotating disc made of parabolically varying functionally graded material. The composite disc made of aluminium matrix reinforced with silicon-carbide particles is taken. The creep analysis is carried out using Sherby's creep law and von Mises' criterion.

2. Reinforcement distribution in the disc

A functionally graded disc made of aluminium matrix reinforced with silicon-carbide particles $Al - SiC_p$, having inner radius a and outer radius b, rotating with angular velocity ω is considered. The density and creep parameters vary along the radial distance as the reinforced silicon carbide particles vary parabolically from the inner to outer radius. Thus, the material parameters of the disc are supposed to be functions of the volume fraction of the constituent materials. The volume content (vol %) of SiC, which is denoted as V(r) at any radius r, is given by:

$$V(r) = A - Br^2, \qquad a \le r \le b \tag{1}$$

where

$$A = \frac{b^2 V_{\max} - a^2 V_{\min}}{b^2 - a^2} \tag{2}$$

and

$$B = \frac{V_{\max} - V_{\min}}{b^2 - a^2} \tag{3}$$

Here $V_{\max} = 30 \ vol\%$ and $V_{\min} = 10 \ vol\%$ are taken as the particle content at the inner radius ($a = 0.03175m$) and outer radius ($b = 0.1524m$), respectively.

The density variation in the composite is expressed using the law of mixtures as

$$\rho(r) = \rho_m + (\rho_d - \rho_m)\frac{V(r)}{100} \tag{4}$$

where $\rho_d = 3210\ kg/m^3$ and $\rho_m = 2713\ kg/m^3$ are the densities of the dispersed silicon carbide particles and matrix alloy, respectively (Clyne and Withers, 1993). From eq (1) and eq (4), we get

$$\rho(r) = \rho_m + (\rho_d - \rho_m)\frac{A - Br^2}{100} \tag{5}$$

The average particle content in the FGM disc is $V_{avg} = 20\ vol\%$, and h is the uniform thickness of the disc, then

$$\int_a^b 2\pi r h V(r)\, dr = V_{avg}\pi h(b^2 - a^2) \tag{6}$$

From eq (1) and eq (6), we get the following relation:

$$V_{avg} = A - \frac{1}{2}B(b^2 + a^2) \tag{7}$$