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Off-axis fatigue behaviour of unidirectional laminates based on a microscale fatigue damage model under different stress ratios

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A fatigue damage micro-model is developed for two-phase composite materials. Constitutive law is derived at the level of fibre and matrix and extended for a damaged lamina using the reformulation of Mori-Tanaka model. A new non-dimensional effective local stress based on a failure criterion which is rewritten in term of average stress of fibre and matrix is coupled with a fatigue damage model for prediction of fatigue damage and fatigue life. The proposed model is able to distinguish between damage in fibre and matrix and also stiffness reduction of the matrix and transverse direction during fatigue loading using three different damage variables. The fatigue behaviour of unidirectional carbon/epoxy and glass/epoxy laminates are studied under off-axis loading conditions. Fatigue life of unidirectional composites is predicted at different stress levels, stress ratios and fibre orientations under tension-tension and tension-compression fatigue. The obtained numerical results from the proposed model are in good agreement with the experimental data.

1. Introduction

Popularity of fibre-reinforce composites are widely increased because of light weight in comparison with high stiffness and strength. Fatigue behaviour of composite structures has been a subject of interest in the last several decades. Fatigue behaviour of laminated composites depends on the type of fibre and matrix, type of reinforcement structure, stacking sequence, environmental condition, boundary and loading conditions, frequency, stress ratio, etc. Initiation and propagation of fatigue damage in layer and interface region and mixing of them are so complicated processes and still remain an open issue [1]. The fatigue analysis of composites thus requires a constitutive relation to consider all aforementioned feature which is very difficult if not impossible. In fact, many different fatigue models just contained some of above mentioned issues [2].

The fatigue behaviours of composite materials are strongly affected by stress ratios and loading conditions. Therefore, it is important to investigate the propagation of fatigue damage and associated fatigue life with respect to stress ratios and loading condition in fibre-reinforce composites. The effect of stress ratio was investigated by many authors. Sims and Brogdon [3] have developed a fatigue failure criterion based on Tsai-Hill criterion [4] by substituting the static strength with fatigue strength functions of the number of fatigue cycles. Rotem and Nelson [5] proposed a fatigue failure criterion to predict the fibre-dominated failure of general laminates regardless of stress ratio. El Kadi and Ellyin [6] proposed a fatigue failure criterion based on input strain energy for tension-tension and tension-compression loading to describe the off-axis fatigue behaviour of a unidirectional E-glass/epoxy composite. Petermann and Plumtree [7] developed a unified fatigue failure criterion as energy based that was associated with the normal and shear stresses in a critical plane to consider the fibre orientation and positive stress ratio effects on the off-axis fatigue behaviour of unidirectional composites. Kawai et al. [8,9] described the master S-N curve by using non-dimensional effective stress based on the Tsai-Hill static failure criterion [4] and developed a phenomenological models [10] which combined with modified non-dimensional effective stress for prediction the off-axis fatigue behaviour of unidirectional CFRP and GFRP composites under constant amplitude cyclic loading conditions. Kawai et al. [11] proposed a model, called the anisomorphic constant fatigue life (CFL) diagram approach, to construct the asymmetric and non-linear CFL diagram for a given composite laminates using the S-N relationship for a particular stress ratio and the static strength in tension and compression. Recently, Kawai and Yano developed a method to predict P-S-N curves by constructing a family of anisomorphic CFL diagram with probability of failure for unidirectional [12] and woven [13] carbon/epoxy laminates. Flore and Wegener

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[14] presented a new phenomenological approach to model the stress ratio effect on fatigue life of carbon and glass fibre reinforce plastics.

It is noted that damage is accumulated in composite materials in three stages: the damage accumulates rapidly in first stage then grows at steady rate. In the last stage, the damage shows a rapid growth rate. The fatigue damage initiation is often disregarded in fatigue models; however, it has a significant effect on fatigue life of unidirectional plies [15]. Therefore, the initiation of matrix damage should be considered to accurately predict the damage. Mao and Mahadevan [16] proposed a mathematical model to consider fatigue damage evolution based on a shape of the damage index in composite materials. The same model have used by several authors to predict fatigue damage in three stages [17,18]. May and Hallet [19,15] proposed a model using cohesive interface element to predict initiation and propagation of fatigue damage. Quaresimin and Carraro et al. [20–22] have performed an in-depth study of damage initiation on unidirectional laminate under Multiaxial fatigue loading. They identified two driving forces for damage evolution in the matrix around the fibre, Local Hydrostatic Stress (LHS) and Local Maximum Principal Stress (LMPS) which predict microcrack initiation. Ma et al. [23] investigated the residual strength behaviour of unidirectional carbon fibre-reinforced composites and proposed a fatigue damage model to predict the fatigue damage history for announcing the fatigue mechanism. Recently, Sevenois et al. [24] proposed a model to predict matrix fatigue crack initiation of unidirectional composites over the whole range of stress ratio. They used a power law to estimate the life to damage initiation and defined two uncoupled equations for tension-tension fatigue based on two damage type and one equation for compression-compression fatigue.

The present work is concerned with the effect of stress ratios on fatigue behaviour of composite materials. The rate of damage propagation in composite materials varies significantly with the stress ratios and stress levels. Although there have been lots of attention due to practical importance of this problem, most of the works have been done at the level of ply [6–14,25]. The constitutive model is constructed at the level of fibre and matrix based on micromechanical approach derived from the Mori-Tanaka [26] method and Benveniste's reformulation [27]. In the proposed model the different rate of damage propagation with non-negative mean stress according to the average stress in fibre and matrix is considered. A new modified non-dimensional effective local stress is generalized to the two-phase composites based on Tsai-Hill criterion for the fibre and matrix and then coupled with a fatigue damage mechanics model to predict fatigue behaviour of unidirectional composites at the micro level. Three different damage variables based on physical observation of fatigue damages [28] are used to distinguish initiation and propagation damage at the fibre and matrix level under uniaxial and multiaxial loading conditions. It is noted that the proposed model is able to consider matrix damage in longitudinal direction and the effect of this damage on stiffness reduction of transverse direction. The predicted results are compared with the available experimental data for several composite materials.

2. Micromechanical based constitutive law

Composites materials usually considered as a two-phase consisting of fibre and matrix [27,29,30]. The fibre denoted by index 'f' is considered as transverse isotropic material and the matrix phase denoted by index 'm' is considered as isotropic material which is reasonable in most case. To determine elastic properties of a lamina, reformulation of Mori-Tanka [26] model by Benveniste [27] is used which describes the behaviour of lamina with respect to the fibre and matrix phases.

On the (on-axis) material coordinate system (1,2) of a composite lamina, the average stress and strain relation can be written using Mori-Tanaka as:

$$\begin{aligned} \varepsilon^*_{12} &= C^*_{12}\sigma_{12} \\ \sigma^*_{12} &= S^*_{12}\varepsilon_{12} \end{aligned} \tag{1}$$

where ε^*_{12} is transformation strain or eigenstrain, σ_{12} is uniform stress and C^*_{12} and S^*_{12} are the effective stiffness and compliance tensor, respectively. The effective stiffness and compliance tensor of the composite using Mori-Tanaka's method is obtained as [27]:

$$\begin{aligned} S^*_{ij} &= (V_m S^m_{ij} + V_f S^f_{ij} H_{ij})(V_m I_{ij} + V_f H_{ij})^{-1} \\ C^*_{ij} &= (V_m C^m_{ij} + V_f C^f_{ij} T_{ij})(V_m I_{ij} + V_f T_{ij})^{-1} \end{aligned} \tag{2}$$

where V_f and V_m are volume fractions of the fibre and matrix.I_{ij} is unit tensor and T_{ij} is defined according to Eshelby's tensor [31] and elastic properties of fibre and matrix as[27]:

$$T_{ij} = \left[I_{ij} + K_{ij} S^m_{ij}(C^f_{ij} - C^m_{ij})\right]^{-1} \tag{3}$$

where I_{ij} is a second order unit tensor and K_{ij} is the second order Eshelby's tensor. Also, H_{ij} is related with T_{ij} and defined as [27]:

$$H_{ij} = C^f_{ij} T_{ij} S^m_{ij} = C^f_{ij}\left[I_{ij} + K_{ij} S^m_{ij}(C^f_{ij} - C^m_{ij})\right]^{-1} S^m_{ij} \tag{4}$$

The Eshelby's tensor is given by [31]:

$$\begin{aligned} K_{ij} &= \begin{bmatrix} K_{11} & K_{12} & 0 \\ K_{21} & K_{22} & 0 \\ 0 & 0 & K_{33} \end{bmatrix} \\ K_{22} &= \frac{5 - 4\upsilon_m}{8(1 - \upsilon_m)} \\ K_{21} &= \frac{\upsilon_m}{2(1 - \upsilon_m)} \\ K_{33} &= \frac{1}{2} \\ K_{11} &= K_{12} = 0 \end{aligned} \tag{5}$$

By substituting Eqs. (5) and (4) into Eq. (2), the symmetric compliance matrix and consequently effective elastic moduli can be written as:

$$\begin{aligned} E_{11} &= \frac{V_f^2(H_{11}H_{22} - H_{12}H_{21}) + V_f V_m(H_{11} + H_{22}) + V_m^2}{(V_m S^m_{11} + V_f H_{11} S^f_{11})(V_m + V_f H_{22}) + V_f V_m H_{21}(S^f_{12} - S^m_{12}) - V_f^2 H_{12} H_{21} S^f_{11}} \\ E_{22} &= \frac{V_f^2(H_{11}H_{22} - H_{12}H_{21}) + V_f V_m(H_{11} + H_{22}) + V_m^2}{(V_m S^m_{22} + V_f H_{22} S^f_{22})(V_m + V_f H_{11}) + V_f V_m H_{12}(S^f_{21} - S^m_{21}) - V_f^2 H_{12} H_{21} S^f_{22}} \\ G_{12} &= \frac{G^f_{12} G_m (V_m + V_f H_{33})}{V_m G^f_{12} + V_f H_{33} G_m} \\ \upsilon_{12} &= -\frac{S_{21}}{S_{11}} \\ &= \frac{(V_m S^m_{12} + V_f H_{11} S^f_{12})(V_m + V_f H_{22}) + V_f V_m H_{21}(S^f_{22} - S^m_{22}) - V_f^2 H_{12} H_{21} S^f_{12}}{(V_m S^m_{11} + V_f H_{11} S^f_{11})(V_m + V_f H_{22}) + V_f V_m H_{21}(S^f_{12} - S^m_{12}) - V_f^2 H_{12} H_{21} S^f_{11}} \end{aligned} \tag{6}$$

where E_{11}, E_{22} are longitudinal and transverse modulous, respectively. G_{12} is shear modulous and υ_{12} is Poisson's ratio.

In order to predict stiffness reduction in the fibre and matrix using the continuum damage mechanics (CDM) concept, three damage variables which represent the loss of stiffness in the fibre and matrix are defines as:

$$D_p = \frac{E_{0,p} - E_{D,p}}{E_{0,p}} \tag{7}$$