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Revisiting energy dissipation due to elastic waves at impact of spheres on large thick plates

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ABSTRACT

In the present study, the model of Hunter (1957) [5] is recalculated using Reed's (1985) [6] more accurate force-time approximation of the perfect elastic impact at which the result of Reed is crucially corrected. The original result from Hunter (commonly known as Hunter loss) underestimates the energy dissipation due to elastic waves in large thick plates by almost 15%. The "corrected Hunter loss" shows an approximately 97% high congruence with the result of FEM simulations of Wu (2001) [7] for the impact between a small steel sphere and a large thick steel plate. Furthermore, the validity of the model of Hunter is discussed.
For a comprehensive estimation of the influence of elastic waves, a large number of experimental impact tests have been carried out at which different material combinations of the impacting bodies have been chosen at low impact velocities (0.5–2.3 m/s). Depending on the material combination and impact velocity, the corrected Hunter loss has a contribution of less than 1% up to about 70% of the total energy dissipation for the performed as well as the evaluated experiments (from literature).

1. Introduction

Several scientific studies regarding elastic waves have to be considered for the description and evaluation of the impact of spheres on large thick plates (or more precisely, an elastic half space) for example, the pioneer works from Lamb [1,2], Miller and Pursey [3,4], in particular the publication from Hunter [5], the critical review from Reed regarding the model of Hunter [6], the FEM (Finite Element Method) simulations from Wu [7,8] as well as the alternative approach from Argatov [9].

The motivation of the present study results from the theoretically determined results of Wu and Argatov [7–9] that show better correlation to the result from Hunter [5] than to the one from Reed [6]. In contrast, experiments and Reed's more accurate approach for the derivation (compared to the one from Hunter) seem to prove the result of Reed. According to the model of Reed [6], elastic waves may have significant influence on the impact process. According to the results of Hunter [5], Wu [7,8] and Argatov [9], the energy dissipation due to elastic waves is rather significantly smaller. Thus, the percentage of energy dissipated by elastic waves away from the impact area has still to be investigated in more detail.

Moreover, very few comprehensive experimental results are available in the literature [1,6,10,11]. For example, the influence of elastic waves at impact processes where the contact follows elastic-plastic and/or viscoelastic mechanics still remains an open question.

If the elastic waves generated during impact (longitudinal and transverse waves below the impact area) are reflected several times within the impact plate, a flexural wave develops and significantly influences the impact process [1,11,12]. This has been theoretically as well as experimentally analyzed and described comprehensively in our recent study [12].

2. Selected models for the elastic impact

2.1. Model of Hertz

The Hertzian theory [13,14,15] describes the quasi-static contact force transmitted during impact by an infinite large thick plate (more precisely, half space) in position of rest onto a sphere

$$F_{Hz}(s_{Hz}) = -k\sqrt{r}s_{Hz}^{3/2} \tag{1}$$

at which r represents the radius of the sphere, $s_{Hz}(t)$ the displacement of both bodies and $k = 4/3E^*$ a measure of the stiffness of the contact and E^* the effective (plane-strain) modulus of elasticity

$$E^* = \left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}\right)^{-1} \approx \frac{E_1}{1-\nu_1^2}, \text{ for } E_2 \gg E_1 \tag{2}$$

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Symbols

$C^k[a, b]$	functions with continuous k-fold derivative in the interval [a, b]
$\mathbb{C}$	set of complex numbers
c_L (m/s)	propagation velocity of longitudinal waves
c_{LP} (m/s)	propagation velocity of quasi-longitudinal waves in thin plates
$c_{L,S}$ (m/s)	propagation velocity of quasi-longitudinal waves in thin bars
c_T (m/s)	propagation velocity of transverse waves
d (m)	diameter
E (J)	energy
E (GPa)	modulus of elasticity
e	coefficient of restitution
F (N)	Force
g (m/s^2)	gravitational constant
H (m)	plate thickness
k (Pa)	constant
m (kg)	mass
$\mathbb{N}_0$	set of non negative integers
$\mathbb{R}$	set of real numbers
r (m)	radius
s (m)	displacement
t (s)	time
v_0 (m/s)	impact velocity
v_F (m/s)	yield velocity
$\mathbb{Z}$	set of integers
Γ	Gamma function
γ	ratio of propagation velocity of a longitudinal and transverse wave
λ	energy dissipation
$\tilde{\lambda}$	energy dissipation
ν	Poisson's ratio
ρ (kg/m^3)	density

Indices

*	effective value
0	initial condition
1, 2	contact partner 1 (sphere) and 2 (plate)
50	mean value
A	Argatov
cor	corrected
diss	dissipated
F	yield
H	Hunter
Hz	Hertz
k	contact
kin	kinetic
max	maximum
meas	measured
min	minimum
N	Newton
R	Reed
W	wave
Wu	Wu

with E_1 and E_2 the moduli of elasticity and ν_1 and ν_2 the Poisson's ratios of the contact partners, respectively, the sphere (index: 1) and the plate (index: 2).

The maximum displacement and indentation, respectively, can be determined by means of the law of conservation of energy

$$s_{Hz,max} = \left(\frac{5}{4}\frac{mv_0^2}{k\sqrt{r}}\right)^{\frac{2}{5}} \tag{3}$$

with the mass m and the impact velocity v_0. The contact time t_k can be calculated numerically using the maximum displacement [15]

$$t_k = \frac{2}{v_0}\int_0^{s_{Hz,max}} \frac{ds_{Hz}}{\sqrt{1-(s_{Hz}/s_{Hz,max})^{5/2}}} = \frac{2s_{Hz,max}}{v_0}\int_0^1 \frac{d\varepsilon}{\sqrt{1-\varepsilon^{5/2}}} = \frac{2.94 s_{Hz,max}}{v_0}. \tag{4}$$

2.2. Model of Miller and Pursey

In a paper published in 1953/54, Miller and Pursey [3] discuss (uniformly) distributed (e.g. circular) loadings and thereby generated elastic waves in an elastic isotropic half space. Based on Lamb's studies [1,2], which only account for (almost) point loadings, the free surface has been normally stimulated by periodic or more precisely, sinusoidal area loadings produced by certain types of electromechanical transducer. It was observed that both the radiation intensity and the material constants have influence on the energy converted into elastic waves. Miller and Pursey had found integrals for the mean displacement over the externally stressed region of the surface and these have been evaluated numerically in a number of cases to obtain the radiation impedance of the sources represented by such a stress system.

In a subsequent publication in 1955 [4], they showed that the maximum energy of seismic waves is accounted for surface waves. Considering a single-element radiator and a Poisson's ratio of $\nu_2 = 0.25$ (e.g. glass), one would find that 67% of the energy is transmitted in the Rayleigh, 26% in the transverse and 7% in the longitudinal wave.

2.3. Model of Hunter

Hunter [5] proved in his paper published in 1957 that in contrast to the Hertzian impact theory, during an elastic collision of a small body with the plane surface of a massive specimen, a small amount of energy dissipation occurs in either case. This energy dissipation is caused by longitudinal, transverse and surface waves which transport a small portion of the impact energy irrecoverably (to infinity), as long as the plate is sufficiently extended and thick so that the waves refracted or reflected at interfaces cannot propagate back to the point of excitation during the time of contact. Hunter's model [5] is based on the quasi-static Hertzian impact theory [13,15] and also incorporates the model of Miller and Pursey [3,4].

The displacement-time function $s_{Hz}(t)$ which is a result of a perfect elastic impact between a sphere and a massive plate is obtained when the normal force-displacement relation according to Hertz (Eq. (1)) is equated with the Newtonian force of a sphere

$$F_N(t) = m\frac{d^2 s_{Hz}}{dt^2}. \tag{5}$$

Since the resulting differential equation can only be numerically calculated, Hunter approximated the displacement-time function by the half period of a simple sine function of the maximum displacement (Eq. (3)) and the Hertzian contact time as follows

$$s_H = s_{Hz,max}\sin\left(\frac{\pi}{2.94}\frac{v_0 t}{s_{Hz,max}}\right). \tag{6}$$

Fig. 1 shows the slight difference between $s_{Hz}(t)$ and $s_H(t)$.

Hunter calculates the force using Newton's second law according to Eq. (5)

$$F_H(t) = \begin{cases} -m s_{Hz,max}\left(\dfrac{\pi v_0}{2.94 s_{Hz,max}}\right)^2 \sin\left(\dfrac{\pi}{2.94}\dfrac{v_0 t}{s_{Hz,max}}\right), & \text{when } 0 \le t \le t_k \\ 0, & \text{else} \end{cases}. \tag{7}$$