International Journal of Mechanical Sciences 123 (2017) 20–33

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Theoretical analysis on the collapse of dumbbell-shaped tubes

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ARTICLE INFO

Keywords:
Energy absorption
Plastic hinge theory
Thin-walled structure
Dumbbell-shaped tube
Self-locked system

ABSTRACT

Thin-walled round tube systems are widely used in impact protection. However, in these systems, additional installation time and cost is required to constrain the tubes and prevent their splashing under impact loadings. To address this problem, a self-locked system comprised of thin-walled dumbbell-shaped tubes was recently proposed, which can prevent lateral splash from impact loadings without the presence of any constraints on the boundary or between the tubes. This paper provides a theoretical analysis on the deformation and collapse of dumbbell-shaped tubes under quasi-static lateral loads. A plastic hinge model is developed to estimate the force-displacement relationship, deformation efficiency and specific energy absorption of the dumbbell-shaped tube, and besides, an elastic solution is derived to describe the mechanical response of the dumbbell-shaped tube at small deformation. The theoretical models are validated by both finite element method (FEM) simulation and experiments. Based on the theoretical analysis, the effects of the geometry of the dumbbell-shaped tube on energy absorption are studied, and the optimal geometry design of the tube is discussed. The relation between each geometry parameter and energy absorption properties is summarized in a table, which provides important reference for designing dumbbell-shaped tubes in practical applications.

1. Introduction

Thin-walled round tube is one of the most widely used structural elements in impact energy absorption due to its low cost and high energy absorption [1–7]. However, round tube system requires internal connections and boundary constraints to suppress the lateral splashing of tubes [8,9]. Complicated constraints can increase installation time and thus make round tube system unsuitable for emergent protection. To break this limitation, a novel self-locked system is recently proposed [10]. The self-locked system is composed of dumbbell-shaped self-locked tubes. When subjected to lateral impact, the dumbbell-shaped tubes can mesh with each other and thus provide lateral constraints to prevent lateral splashing. Therefore, the self-locked system can work immediately after the tubes are put together, with no need of any fasteners between tubes or boundary constraints. In literature [10], the quasi-static compression of a dumbbell-shaped tube between two rigid plates has been studied by finite element method (FEM) simulation, while a theoretical analysis is still in demand to provide comprehensive study on the mechanical behavior of the dumbbell-shaped tubes and to reveal the effects of geometry parameters on the energy absorption.

Many researchers have studied the plastic deformation of thin-walled round tubes with the plastic hinge theory. In this theory, the localized plastic deformation zones are idealized as plastic hinges while elastic deformations are neglected. For the quasi-static lateral compression of a tube between two rigid plates, DeRuntz and Hodge [11] developed a rigid perfect plastic four hinge model to determine the compressing load beyond initial yield. Their model can give an approximate prediction of the load-displacement curve until the top hinge reaches the bottom one. While DeRuntz and Hodge's model cannot fully flatten the tube, Burton and Craig [12] proposed a six hinge model and addressed this problem. Redwood [13] found that neglecting the strain hardening effect would underestimated the crushing load especially at large deformation and improved DeRuntz and Hodge's model by including the effect of linear strain hardening. Besides the quasi-static lateral compression of a round tube between rigid plates, the plastic hinge theory has also been applied to analyze the lateral deformation of tubes and tube systems with various cross-sections and loading conditions. Reddy and Reid [14] studied the lateral crushing of a round tube with side constraints. Yu [15] analyzed the plastic deformation of a ring pulled by diametrical point loads with a six hinge model. Ghosh, et al. [16] studied the diametrical crushing of tubes under concentrated loads. Shim and Stronge [17] studied the tubes crushed by cylindrical indenters. Shen, et al. [18] proposed different plastic hinge models for sandwich tubes with different

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http://dx.doi.org/10.1016/j.ijmecsci.2017.01.031
Received 6 October 2016; Received in revised form 24 December 2016; Accepted 17 January 2017
Available online 22 January 2017
0020-7403/

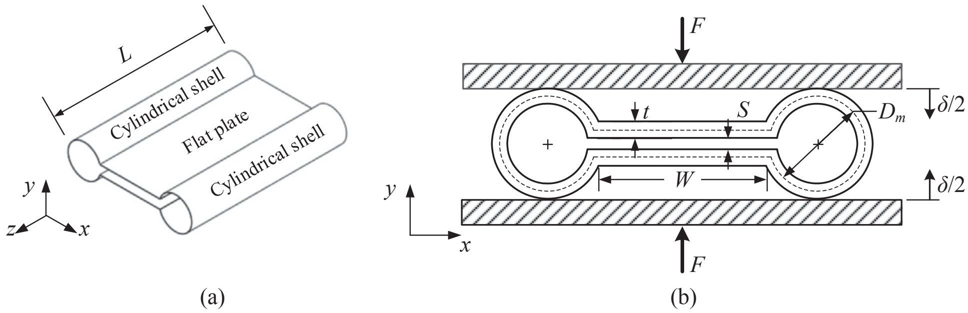

Fig. 1. Schematic plots of (a) a dumbbell-shaped tube and (b) the cross section of the tube with geometry parameters and loading condition.

collapse modes. Lipa and Kotełko [19] derived the impact response of a one dimensional tube system with four hinge model. Wang et al. [20,21] developed plastic hinge models to analyze the lateral impacting of internally nested round tube systems and elliptical tubes.

In this work, theoretical models are developed to study the static compression of a dumbbell-shaped tube between two parallel rigid plates. Section 2 outlines the formulation of theoretical models. An elastic solution based on small deformation assumption is obtained to describe the elastic response of the dumbbell-shaped tube. A plastic hinge model is developed to model the plastic crushing of dumbbell-shaped tube with the effect of linear strain hardening. In Section 3, the results of theoretical prediction are validated against experiment and FEM simulation. In Section 4, the theoretical models are applied to analyze the effects of geometry parameters on energy absorption. Conclusion is provided in Section 5.

2. Theoretical analysis

The dumbbell-shaped tube is comprised of two open cylindrical shells and two parallel flat plates, as shown in Fig. 1(a). The tube cross-section and loading conditions are plotted in Fig. 1(b). The geometry of dumbbell-shaped tube is determined by five design parameters: the length of the tube L, the wall thickness t, the spacing of the flat plate S, the width of the flat plate W and the mean diameter D_m. Here the mean diameter D_m is the average of the outer and inner diameters of the cylindrical shell. The tube is laterally compressed by two parallel rigid plates. The compressive force is F and the vertical relative crushing displacement between the upper and lower loading plates is δ. Because the length L is significantly larger than the dimensions of cross section, the deformation of the tube is considered under plane strain condition [11,22]. The tube first has a short elastic deformation process and then goes into elastic-plastic process. Accordingly, an elastic solution at small deformation is provided in Section 2.1. Then, the elastic-plastic crushing is analyzed under rigid linear hardening plastic assumption in Section 2.2, since it has been noted in literature [11] that the elastic strain is negligible in comparison to the deformation of the tube. Due to the symmetry of geometry and loading arrangement, only a quarter of the tube is considered in the following analysis.

2.1. Linear elastic analysis for small deformation

In elastic stage, the crushing of dumbbell-shaped tube can be analyzed with the linear elastic beam model as depicted in Fig. 2. Due to symmetry and continuity conditions, the curved beam ABCD is assumed to be clamped on the horizontal symmetry plane at point D and the rotation at point A is constrained by moment M_A (See Appendix A). The crushing load $F/2$ is located at the upper extremity of the vertical diameter point C. The elastic strain energy in bending is

$$U = \int_{l_{\widehat{ABC}}} \frac{M_A^2}{2EI} \cdot dl + \int_0^{\frac{\pi}{2}} \frac{\left(M_A - \frac{F}{2}\cdot\frac{D_m}{2}\cdot\sin\theta\right)^2}{2EI}\cdot\frac{D_m}{2}\cdot d\theta \tag{1}$$

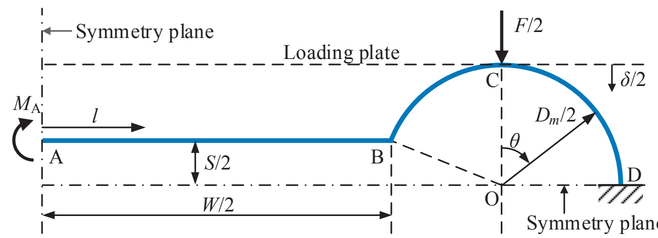

Fig. 2. Linear elastic beam model of the dumbbell-shaped tube under crushing.

where $l_{\widehat{ABC}} = W/2 + (D_m/2)[\pi/2 - \arcsin(S/D_m)]$ is the length of arc $\widehat{ABC}$, E is the Young's modulus, $I = Lt^3/12$ is the moment of inertia and l is the intrinsic coordinate starting from point A. According to the theorem of Castigliano [23], the rotation angle at point A, which is zero due to symmetry and continuity conditions, is expressed as the partial derivative of elastic strain energy U with respect to moment M_A

$$\frac{\partial U}{\partial M_A} = 0 \tag{2}$$

From Eqs. (1) and (2), we have

$$M_A = \frac{{D_m}^2}{4W + 4\pi D_m - 4D_m \arcsin\frac{S}{D_m}} \cdot F \tag{3}$$

The crushing displacement with respect to the horizontal symmetry plane is

$$\frac{\delta}{2} = \frac{\partial U}{\partial\left(\frac{F}{2}\right)} \tag{4}$$

From Eqs. (1), (3) and (4), the total crushing load of a dumbbell-shaped tube is obtained as

$$F = \frac{8}{3}\frac{E\hat{t}^3 L D_m}{\pi - \frac{4}{\hat{W} + (\pi - \arcsin\hat{S})}} \cdot \hat{\delta}, \tag{5}$$

in which $\hat{t} = t/D_m$, $\widehat{W} = W/D_m$ and $\hat{S} = S/D_m$ are the normalized geometry parameters of the tube, and $\hat{\delta} = \delta/D_m$ is the normalized displacement of the loading plate.

It is clearly indicated in Eq. (5) that the crushing force F increases with the normalized thickness $\hat{t}$, the normalized plate spacing $\hat{S}$, the tube length L and the mean diameter D_m, and decreases with the normalized plate width $\widehat{W}$.

The FEM simulation is carried out to validate the theoretical model, with simulation details provided in Appendix B and results presented in Fig. 3. It is found from Fig. 3 that the elastic solution in Eq. (5) agrees well with the simulation results when the normalized displacement $\hat{\delta}$ is less than 0.05. But if $\hat{\delta}$ becomes larger, some part of the tube material goes into plastic stage. Accordingly, the elastic model is no longer proper for this problem and a plastic analysis is in demand.