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Vibration of a multilayer graphene sheet under layerwise tension forces

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ABSTRACT

The present work is inspired by the fact that tension enforced by external force applied directly to the outermost layers of a (usually incommensurate) multilayer graphene sheet cannot be effectively transferred to all inner layers due to interlayer sliding, and therefore tension force in inner layers can be much lower than the tension force in the outermost layers. In this paper, a three-beam model is presented to study vibration of a multilayer graphene sheet under layerwise tension forces. In contrast to the commonly used single-beam model which assumes that tension in all layers of a multilayer graphene sheet are identical, the present model treats the top and bottom layers as two beams, and all other inner layers together as another beam which has different tension force than the top and bottom beams. Our results indicate that actual tensile stress/strain in the outmost singlelayers of a multilayer graphene sheet can be much (for instance, almost ten times, for specific examples discussed here) higher than that estimated by the widely used single-beam model, and the latter can badly underestimate actual tensile stress/strain of multilayer graphene resonators. In addition, at least for typical examples discussed here, the present model shows that vibrational frequencies of a multilayer graphene sheet are largely determined by the total tension, and the distribution of the total tension over different layers does not make a huge impact to vibrational frequencies of multilayer graphene resonators. Based on this conclusion, an explicit formula is given for resonant frequencies of multilayer graphene sheets under layerwise tension forces although the actual maximum tension depends on how total tension is distributed over all layers.

1. Introduction

Graphene [1] has attracted widespread attention since its discovery. Despite being only one atom thick, graphene sheets (GSs) exhibit superior physical, electronic and mechanical properties [2–4], such as high flexibility, high stiffness, low mass, high electrical and thermal conductivity. Recently, mechanical resonance of graphene has been studied with various experimental methods, such as optical interferometry [5–7], or electrostatical imaging by scanning force microscopy [8]. Vibration of suspended GSs is measured at room temperature by Bunch et al. [5]. For total 33 GSs of thickness ranging from singlelayer to 75 nm, as summarized in Fig. 3 of [5], the measured fundamental frequency varies from 1 MHz to 170 MHz. In their analysis, classical single-beam model was used to simulate vibration of doubly clamped or cantilever GSs based on an assumption that the measured GSs are under tension, which dominates the fundamental resonant frequency. Similar phenomena were observed by Takamuraet al. [7]. For example, for a trilayer graphene of thickness 1 nm and with length 7.6 μm and width 3.7 μm, experimental fundamental frequency [7] is 7.52 MHz. If the classical single-beam formula with zero tension force is used, a value of 0.37 MHz would be obtained, which is much lower than their measured value. This example clearly indicates that tension force plays a dominant role in vibration of stretched GSs. Frank et al. [9] measured effective spring constant of stacks of GSs suspended over photolithographically defined trenches in silicon dioxide by using an atomic force microscope. When their data [9] are fitted to the doubly clamped classical single-beam model under tension, they extracted a Young's modulus of 0.5TPa and a total tension force on the order of 10^{-7}N. Traversi et al. [10] measured the mechanical characterization of suspended GSs by nanoindentation with an atomic force microscopy tip. Fitting the doubly clamped classical single-beam model to their [10] experiment data, they obtained a built-in tension of 12nN and a Young's modulus of 0.43TPa. All of the mentioned previous works have well confirmed the validity and accuracy of the classical elastic beam model (without so-called "non-local effects") for tension-dominated vibration of GSs under various end conditions.

Previous researches on vibration of multilayer graphene sheets (MLGSs) are all based on the single elastic beam model which assumed that tension forces in all layers of a MLGS are identical. In reality,

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multilayer graphene grown by various available techniques are interlayer misoriented in most cases, called "incommensurate", with vanishingly weak interlayer in-plane coupling. As a consequence, it is well-known [25–28] that the incommensurability leads to ultralow interlayer (both static and kinetic) friction. Typically, for example, it has been repeatedly reported in literature that the interlayer friction coefficient for incommensurate multilayer garphene is on the order of 0.001 [25]. Therefore, due to interlayer sliding [11–13,25–28], it is expected that tension enforced by external force applied to the outermost top and bottom layers cannot be effectively transferred to all inner layers, and therefore tension force in all inner layers can be much lower than the tension in the top and bottom layers. This crucial feature of stretched MLGSs has not been addressed in existing literature. It is this key shortcoming of all previous related models that motivates the present work. Unlike the widely used single-beam model which assumes that tension forces in all layers of a MLGS are identical, the present work aims to study free vibration of a MLGS under layerwise tension forces.

In this paper, a simple three-beam model is presented for vibration of a MLGS. A MLGS is modeled as a simplified three-beam system in which the top and bottom layers are modeled as two individual beams, and all other un-tensioned or less-tensioned inner layers are modeled as another beam, and the three beams are coupled through van der Waals interaction between any two adjacent beams. In particular, the inner beam has a different tension than the top and bottom beams. The major goal of the present work is to examine the effect of layerwise tension forces on vibrational frequencies of a MLGS. For our goal, the simple classical beam model is appropriate to highlight the role of layerwise tension forces in vibration of a multilayer graphene sheet.

2. The present model

Deflection $w(x,t)$ of an elastic beam under transverse distributed pressure $p(x)$ and axial force F is governed by

$$p(x) + F\frac{\partial^2 w}{\partial x^2} = EI\frac{\partial^4 w}{\partial x^4} + \rho A\frac{\partial^2 w}{\partial t^2} \tag{1}$$

where x is the axial coordinate, t is time, EI and A are the bending rigidity and the cross section area of the beam, and ρ is the mass density per unit volume.

The present paper studies linearized small-amplitude vibration of MLGS. In this paper, a MLGS is modeled as a simplified three-beam system in which the top and bottom layers are modeled as two beams, and all other inner layers are modeled as another single beam, and the three beams are coupled through van der Waals interaction between any two adjacent beams. Thus, the above beam model (1) is applied to the three beams, respectively, in which $p(x)$ represents the van der Waals force between adjacent beams, F represents the tension force. In doing so, the governing equation for free vibration of a MLGS can be given as follows

$$\begin{cases} EI_1\frac{\partial^4 w_1}{\partial x^4} - F_1\frac{\partial^2 w_1}{\partial x^2} + \rho A_1\frac{\partial^2 w_1}{\partial t^2} = bc(w_2 - w_1) \\ EI_2\frac{\partial^4 w_2}{\partial x^4} - F_2\frac{\partial^2 w_2}{\partial x^2} + \rho A_2\frac{\partial^2 w_2}{\partial t^2} = bc(w_1 - w_2) + bc(w_3 - w_2) \\ EI_3\frac{\partial^4 w_3}{\partial x^4} - F_3\frac{\partial^2 w_3}{\partial x^2} + \rho A_3\frac{\partial^2 w_3}{\partial t^2} = bc(w_2 - w_3) \end{cases} \tag{2}$$

where the subscript 1, 2 and 3 denote the top layer, the inner layers, and the bottom layer, respectively, and b is the width of the beams, c is the van der Waals interaction coefficient per unit area between two adjacent beams and can be expressed as [14]:

$$c = \frac{320\times \mathrm{erg/cm^2}}{0.16d^2} \quad (d = 1.42\times 10^{-8}\mathrm{cm}) \\ = 99\ \mathrm{GPa/nm} \tag{3}$$

Here, it should be stated that, to justify the efficiency of the present three-beam model, a comparison between the present three-beam model and more complicated four-beam or five-beam model is made in Table 1 and Table 2. It is shown in Tables 1, 2 that no meaningful difference exists in the results given by the present three-beam model and more complicated multiple-beam models. In other words, in spite of its mathematical simplicity, the present simple three-beam model can catch major features of layerwise tension forces in vibration of a MLGS.

Table 1
Distribution of the tension force of a five-layered GS given by different multiple-beam models.

F_i (tension in ith layer)	F_1	F_2	F_3	F_4	F_5
Three-beam model	F_1	0.5 F_1	F_1	–	–
Four-beam model	F_1	0.3 F_1	0.2 F_1	F_1	–
Five-beam model	F_1	0.2 F_1	0.1F_1	0.2 F_1	F_1

Table 2
Tension force F_1 in the top layer of a simply supported five-layered GS with length 3 µm, width 0.6 µm, predicted by different multiple-beam models. (Fundamental resonant frequency is assumed to be 30 MHz).

	Three-beam model	Four-beam model	Five-beam model
F_1 (nN)	29.0799	29.0799	29.0799

3. Solution procedure

For free vibration, the general solution of 3 coupled fourth-order Eq. (2) is a linear combination of 12 independent particular solutions of the homogeneous equations. The particular solutions for the displacements of the top layer, the inner layers and the bottom layer can be represented by

$$w_j(x,t) = Y_j(x)e^{i\omega t} = B_j e^{\lambda x + i\omega t}, \quad (j = 1, 2, 3) \tag{4}$$

where $i = \sqrt{-1}$, ω is resonant frequency of the MLGS, and B_j are unknown coefficients. Substituting expressions (4) into Eq. (2), yields a matrix equation

$$\begin{pmatrix} EI_1\lambda^4 - F_1\lambda^2 - \rho A_1\omega^2 + bc & -bc & 0 \\ -bc & EI_2\lambda^4 - F_2\lambda^2 - \rho A_2\omega^2 + 2bc & -bc \\ 0 & -bc & EI_3\lambda^4 - F_3\lambda^2 - \rho A_3\omega^2 + bc \end{pmatrix} \times \begin{pmatrix} B_1 \\ B_2 \\ B_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{5}$$

The associated amplitude ratio of the three beam model can be determined as

$$\frac{B_2}{B_1} = 1 + \frac{EI_1\lambda^4 - \rho A_1\omega^2 - F_1\lambda^2}{bc} \tag{6}$$

$$\frac{B_3}{B_1} = \frac{EI_1\lambda^4 - F_1\lambda^2 - \rho A_1\omega^2 + bc}{EI_3\lambda^4 - F_3\lambda^2 - \rho A_3\omega^2 + bc} \tag{7}$$

Accordingly, the existence condition for a nonzero solution B_j (j=1,2,3) of Eq. (5) leads to a twelve order algebraic equation for the eigenvalue λ, which determine 12 eigen-roots. Thus, the general solution of (2) is a linear combination of the 12 particular solutions with 12 arbitrary coefficients. Substituting the general solution to total 12 end conditions of the 3 elastic beams leads to 12 homogeneous equations for the 12 non-zero coefficients. The existence condition for the 12 non-zero coefficients leads to the condition to determine the