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Flow behind magnetogasdynamic exponential shock wave in self-gravitating gas

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A B S T R A C T

The propagation of a cylindrical (or spherical) shock wave driven out by a piston moving with time according to an exponential law, in a self-gravitating ideal gas with azimuthal magnetic field is investigated. The initial magnetic field is assumed to be varying according to an exponential law. Solutions are obtained for both the cases of isothermal and adiabatic flows. The effects of variation of ambient magnetic field, gravitational parameter and adiabatic exponent are worked out in detail. It is manifested that the increase in strength of ambient magnetic field has decaying effect on the shock wave however increase in the value of gravitational parameter has reverse effect on the shock strength. The compressibility of the medium is increased in the presence of gravitational field. Also, a comparison between the solutions obtained in the case of isothermal and adiabatic flows is done. Density, pressure, velocity and magnetic field increases, however mass decreases as we move inward from the shock front towards the piston.

1. Introduction

Shock waves have applications in astrophysics, plasma physics, geophysics, nuclear science and interstellar masses for non-linear systems and in many other fields. Shock waves are common in the interstellar medium because of a great variety of supersonic motions and energetic events, such as cloud-cloud collision, bipolar outflow from young protostellar objects, powerful mass losses by massive stars in a late stage of their evolution (stellar winds), supernova explosions, central part of star burst galaxies, etc. These waves are also associated with quasars, radio galaxies and spiral density waves. Similar phenomena also occurs in laboratory situations, for example, when a piston is driven rapidly into a tube of gas (a shock tube), when a projectile or aircraft moves supersonically through the atmosphere, in the blast wave produced by a strong explosion, or when rapidly flowing gas encounters a constriction in a flow channel or runs into a wall (Nath and Vishwakarma [1]). Shock phenomena like a global shock which results from a supernova explosion or stellar pulsation or a shock arising from a point source, have appalling importance in space science. Shocks are prevalent throughout the observed universe and play a decisive role in the processes observed in nebulae that eventually could lead to formation of new stars. Shock waves produced by solar flares represent the most extreme manifestation of solar activity (Nath [2]). According to observational data, the unsteady motion of a large mass of gas followed by sudden release of energy results in flare-ups in novae and supernovae. A qualitative behavior of the gaseous mass may be discussed with the help of the equations of motion and equilibrium taking gravitational forces into account. Sedov [3] and Carrus et al. [4] independently derived numerical solutions for adiabatic flows in self-gravitating gas. Purohit [5] and Singh and Vishwakarma [6] have studied homothermal flows behind a spherical shock wave in a self-gravitating case. Nath et al. [7] obtained the consequences of presence of magnetic field for the above problem, considering the flow to be self-similar and adiabatic. Many investigators studied the interaction between gasdynamic motion of an electrically conducting medium and magnetic field within the context of hyperbolic system (Korobeinikov [8], Shang [9] and Lock and Mestel [10]).

Magnetic fields permeate the universe and have crucial roles in a number of astrophysical situations. All astrophysical plasmas are likely affected by magnetic fields. Magnetic fields play an important role in energy and momentum transport and can rapidly release energy in flares. Many interesting problems involve magnetic fields. The shock waves in the presence of a magnetic field in conducting perfect gas can be important for description of shocks in supernova explosion and explosion in the ionosphere. Complex filamentary structures in molecular clouds, shapes and the shaping of planetary nebulae, synchrotron radiation from supernova remnants, magnetized stellar winds, galactic winds, gamma-ray bursts, dynamo effects in stars, galaxies, and galaxy clusters as well as other interesting problems all involve magnetic fields. The industrial applications are drag reduction in duct flows,

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design of efficient coolant blankets in tokamak fusion reactors, control of turbulence of immersed jets in the steel casting process and advanced propulsion and flow control schemes for hypersonic vehicles, involving applied external magnetic fields (see Hartmann [11], Balick and Frank [12]).

In all the above mentioned works, the effects of self-gravitation are not taken into account by any of the authors for the case of an exponential shock for isothermal and adiabatic flows with magnetic field. The motion of piston is assumed to obey an exponential law, namely (Ranga Rao and Ramana [13], Singh and Vishwakarma [14])

$$r_p = A\exp(\alpha t); \quad \alpha > 0, \tag{1}$$

where r_p is radius of the piston, A and α are dimensional constants, and t is the time. 'A' represents the initial radius of the piston. It, may be, physically, the radius of the stellar corona or the condensed explosives or the diaphragm containing a very high-pressure driver gas, at t=0. By sudden expansion of the stellar corona or the detonation products or the driver gas into the undisturbed ambient gas, a shock wave is produced in the ambient gas. The shocked gas is separated from the expanding surface which is a contact discontinuity. This contact surface acts as a 'piston' for the shock wave in the ambient medium [15–17].

Eq. (1) implies a boundary condition on the gas speed at the piston. Since we have assumed self-similarity, we may postulate that

$$R = B\exp(\alpha t), \tag{2}$$

where R is shock radius, and B is a dimensional constant. B depends on A and non-dimensional position of the piston. The 'piston' is used to replicate blast waves and other similar phenomena in a model in order to simulate actual explosions and their effects, usually on a smaller scale. Thus 'piston' problem can be applied to quantify an estimate for the outcome from supernova explosions, sudden expansion of the stellar corona or detonation products, central part of star burst galaxies etc.

In this work, we extended the solution obtained by Ranga Rao and Ramana [13] to the case of magnetogasdynamic in a self-gravitating gas. We have obtained self-similar solutions for the propagation of a cylindrical (or spherical) strong shock wave generated by a moving piston according to an exponential law in a self-gravitating ideal gas under the influence of magnetic field, under isothermal and adiabatic flow conditions. It is necessary to take the density of the ambient medium to be constant for obtaining similarity solutions of the problem. The media ahead and behind the shock front is assumed to be inviscid, to behave as thermally perfect gas and the magnetic field in the ambient medium vary exponentially.

The effects of an increase in the value of gravitational parameter, adiabatic exponent and the strength of ambient magnetic field are worked out in detail. It is shown that the density ratio β increases on increasing strength of ambient magnetic field and value of adiabatic exponent i.e. the shock strength decreases. Compressibility of the medium is increased in the presence of gravitation. The distance between the piston and the shock front is less in case of adiabatic flow that in the case of isothermal flow but reverse effect is observed for high values of gravitational parameter G_0. Ambient magnetic field and gravitation have opposite effects on shock strength.

2. Fundamental equations and boundary conditions: isothermal flow

The equations of motion for unsteady and cylindrically (or spherically) symmetric isothermal flow of an electrically conducting self-gravitating ideal gas, in the presence of an azimuthal magnetic field and neglecting the viscosity, have the form (cf. Carrus et al. [4], Whitham [18], Laumbach and Probstein [19], Sachdev and Ashraf [20], Korobeinikov [8], Zhuravskaya and Levin [21]), Nath and Sinha [22], Nath et al. [23])

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{iu\rho}{r} = 0, \tag{3}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\left[\frac{\partial p}{\partial r} + \mu h\frac{\partial h}{\partial r} + \frac{\mu h^2}{r}\right] + \frac{Gm}{r^i} = 0, \tag{4}$$

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial r} + h\frac{\partial u}{\partial r} + (i-1)\frac{hu}{r} = 0, \tag{5}$$

$$\frac{\partial m}{\partial r} = 2\pi i\rho r^i, \tag{6}$$

$$\frac{\partial T}{\partial r} = 0, \tag{7}$$

where t and r are independent time and space coordinates, ρ the density, u the fluid velocity, p the pressure, h the azimuthal magnetic field, T the temperature, μ the magnetic permeability, m the mass, i=1, 2 for the respective cases of cylindrical and spherical symmetries, and G the gravitational constant.

The above system of equations should be augmented with an equation of state. As the behavior of the medium is supposed to be ideal, so that

$$p = \Gamma\rho T \quad ; \; U_m = \frac{p}{(\gamma-1)\rho}, \tag{8}$$

where Γ is the gas constant, and γ is the adiabatic index.

A strong shock wave is supposed to be propagating in the undisturbed ideal gas with constant density ρ_1 in the presence of an azimuthal magnetic field varying as $h_1 = h_0\exp(-\lambda t)$, where h_0 and λ are constants.

The flow variables just ahead of the shock front are

$$u_1 = 0,\ \rho = \rho_1 = constant, m = m_1 = \frac{2\pi i\rho_1 R^{i+1}}{(i+1)},\ h = h_1 = h_0\exp(-\lambda t). \tag{9}$$

The jump conditions for magnetogasdynamic shock wave are given as(cf. Nath and Sinha [22], Nath et al [23])

$$\rho_1 C = \rho_2(C-u_2),\ h_1 C = h_2(C-u_2),$$
$$p_1 + \frac{1}{2}\mu h_1^2 + \rho_1 C^2 = p_2 + \frac{1}{2}\mu h_2^2 + \rho_2(C-u_2)^2,\ U_1 + \frac{p_1}{\rho_1}$$
$$+\frac{1}{2}C^2 + \frac{\mu h_1^2}{\rho_1} - \frac{q_1}{\rho_1 C} = U_2 + \frac{p_2}{\rho_2} + \frac{1}{2}(C-u_2)^2 + \frac{\mu h_2^2}{\rho_2} - \frac{q_2}{\rho_1 C},$$
$$m_1 = m_2, \tag{10}$$

where $C = (\frac{dR}{dt})$ denotes the velocity of the shock front, and 'q' is the radiation heat flux.

Since the shock is strong, therefore

$$p_1 \approx 0, \quad U_{m_1} \approx 0. \tag{11}$$

From Eqs. (10), the conditions across a strong shock propagating into an ideal gas reduce to,

$$u_2 = (1-\beta)C,\ \rho_2 = \frac{\rho_1}{\beta},\ p_2 = \left[(1-\beta) + \frac{1}{2M_A^2}\left(1-\frac{1}{\beta^2}\right)\right]\rho_1 C^2,\ m_2 = m_1,\ h_2 = \frac{h_1}{\beta}, \tag{12}$$

where $M_A = \left(\frac{\rho_1 C^2}{\mu h_1^2}\right)^{\frac{1}{2}}$ is the Alfven-Mach Number.

The density ratio $\beta\,(0<\beta<1)$ across the shock front is obtained from,

$$\frac{\beta[1+\beta(\gamma-1)]}{\gamma-1} = \frac{F_2 - F_1}{Cp_2} + \frac{(1-\beta^2)}{2}. \tag{13}$$

As the shock is strong, $(F_2 - F_1)$ is assumed to be negligible in comparison to the product of p_2 and C. Therefore, Eq. (13) reduces to the quadratic equation,