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Multiply heaving bodies in the time-domain: Symmetry and complex resonances

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ABSTRACT

The time-domain linear water wave problem for multiple bodies which are constrained to move in heave after being given an arbitrary initial displacement is considered. It is shown that the solution can be calculated in a simple and efficient method using the frequency domain solution. The resonant behaviour of the system is then investigated and an approximation to the solution using the singularity expansion method derived. A method to exploit any symmetry in the multiple body geometry, which makes the calculations more efficient and simplifies the analysis, is presented. Numerical calculations are performed for truncated cylinders, using bespoke code that computes the solution for complex frequencies. A method to approximate the solution for complex frequencies using results for real frequencies is derived and this is used to make calculations for heaving hemispheres using existing general purpose computer code.

1. Introduction

The linear wave-structure interaction problem is one of the best studied problems in hydrodynamics and it is the basis for numerous practical applications. Generally the solution is found by assuming the response is at a single frequency. The solution for more complex forcing or for time-dependent motion is then found using the superposition property of linearity. A well known example of these kinds of methods is the *Memory effect* (or *Cummins*) *method* (Cummins, 1962; Mei, 1989) which calculates the time-dependent response by a time-stepping convolution integral with a kernel function which may be calculated from the frequency-domain solutions, or calculated directly. In a recent work Meylan (2014) developed a method to calculate the solution for the time-domain problem in which the solution for all times is calculated without the need for any time-stepping or convolution integral. This method is based upon that given by Ursell (1964), Maskell and Ursell (1970) for the very simplest hydrodynamic problem, a half submerged horizontal circular cylinder in water of infinite depth. One key point missed in this previous work was that the integral consists of two parts, one of which is rapidly convergent, while the other is very slowly convergent. This meant that the numerical calculation performed in Ursell (1964), Maskell and Ursell (1970) was inefficient unless the contour of integration was deformed. This contour deformation in turn required that the solution for real frequencies be analytically extended to complex frequencies. This made the method very difficult to generalize. However, by deforming the contour of integration Ursell (1964), Maskell and Ursell (1970) found a way to approximate the solution as a projection onto a complex resonance. This approximation is known as the *Singularity Expansion Method* (SEM).

We present here an extension of the time-domain method developed in Meylan (2014) to three dimensions and to multiple

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floating bodies, restricted to move in heave only. Such arrays of floating bodies have been extensively studied in the frequency domain. In particular, arrays of floating truncated cylinders have been analysed using various semi-analytical methods (McIver, 1984; Mavrakos, 1991; Kagemoto and Yue, 1986; Siddorn and Eatock Taylor, 2008) though only the last of these solved both the radiation and diffraction problems for cylinders moving independently. Arrays of truncated cylinders are of interest as they may be simple models of wave energy device arrays, floating airport pontoons, etc. The related problem in the time domain has received less attention.

The SEM was used by Ursell (1964), Maskell and Ursell (1970) but represents only one very special case. This method is known in the wider literature and has a significant history in electromagnetics (Baum, 1976) and was applied to floating bodies by Beale (1977). The method was developed recently for floating elastic plates (Meylan, 2002; Hazard and Loret, 2007) and for fixed bodies (Meylan and EatockTaylor, 2009; Meylan and Fitzgerald, 2014). For the arbitrary single body (in two dimensions) the method was developed by Meylan (2014). The SEM can be seen as a generalization of the eigenfunction expansion for compact operators. The key difficulty with the SEM is that the solution needs to be extended analytically to complex frequencies. We discuss in detail how this can be achieved and also present an approximate method which allows the complex solution to be estimated from the solution for real frequencies only (Meylan and Tomic, 2012).

Symmetry plays a key role in mathematics and in the water wave context it can be exploited to simplify calculations. For our case of floating arrays the symmetry allows us to transform the problem to block diagonal form, and in some cases to diagonal form. As well as giving insights into the problem, this allows us to simplify our computations and for the kinds of calculations performed here this simplification is of great benefit. The key results come from group theory and they say that if our matrices for added mass, damping, etc commute with the symmetries of the group then the solution must be decomposable into symmetries. This kind of decomposition is commonly applied to compact operators, for example to the eigenfunctions of a compact operator on a symmetric domain. We show that the same decomposition applies here and show how to derive the appropriate change of basis matrices.

The outline of this paper is as follows. In Section 2 we present the equations of motion in the time domain. In Section 3 we show how the solution to these equations can be found using a Fourier transform and written as an integral over frequency-domain solutions (for real frequencies). Section 4 gives an approximate solution for complex resonances using the SEM and Section 5 gives a further approximation of this using the solution for real frequencies to estimate the complex frequency solution. Section 6 is a detailed discussion of the role of symmetry with the examples focused on the symmetry group of a square. In Section 7 we present some numerical calculations for both cylinders and hemispheres and Section 8 contains brief conclusions.

2. Equations of motion in the time-domain

We consider N identical bodies which are able to move in heave only. The linearized equations of motion are written in terms of a velocity potential Φ. The governing equation and boundary conditions are

$$\nabla^2\Phi = 0, \quad \mathbf{x} \in \Omega, \tag{1a}$$

$$\partial_z\Phi = 0, \quad z = -h, \tag{1b}$$

$$\partial_z\Phi + \frac{1}{g}\partial_t^2\Phi = 0, \quad \mathbf{x} \in \partial\Omega_F, \tag{1c}$$

where Ω is the fluid bounded by a free surface $\partial\Omega_F$ at z=0 and the fluid bottom at $z = -h$, while $\mathbf{x}$ is the three-dimensional vector with components (x, y, z) and g is the acceleration due to gravity. We also have a kinematic condition on the surface of the cylinders which is

$$\partial_n\Phi = n_p\partial_t\zeta_p, \quad \mathbf{x} \in \partial\Omega_{B_p} \tag{1d}$$

where ∂_n is the outward normal derivative, n_p is the normal component associated with a unit cylinder heave motion, ζ_p and $\partial\Omega_{B_p}$ are the heave displacement and wetted surface, respectively, of the pth body. Finally, we also have the dynamic condition given by

$$M_{pp}\partial_t^2\zeta_p = -\rho\iint_{\partial\Omega_{B_p}}\partial_t\Phi\, n_p\, \mathrm{d}S - C_{pp}\zeta_p, \quad p = 1\ldots N. \tag{2}$$

where M_{pp} is the mass of body p and C_{pp} is the hydrostatic restoring force coefficient (we do not consider moorings).

We also need to specify initial conditions. We assume that at t=0 the fluid is initially at rest and the bodies are given an initial displacement with no initial velocity. The case of an initial velocity is not difficult to include but is non-physical because it is not possible to excite the body in this way without exciting the surrounding fluid. We denote the initial displacement by $\zeta_p(0)$.

3. Fourier transform solution

We solve the system of equations using the Fourier transform. This is based on the method proposed by Ursell (1964), Maskell and Ursell (1970) for the case of a horizontal circular cylinder in two-dimensions and extended to arbitrary bodies in two-dimensions by Meylan (2014). We define the one sided Fourier transform as

$$\widehat{\zeta_p}(s) = \int_0^\infty \zeta_p(t)\mathrm{e}^{\mathrm{i}st}\,\mathrm{d}t \quad \text{and} \quad \widehat{\Phi}(s) = \int_0^\infty \Phi(t)\mathrm{e}^{\mathrm{i}st}\,\mathrm{d}t. \tag{3}$$