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Calculating fractal parameters from low-resolution terrain profiles

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Abstract

Driver comfort on rough terrain is an important factor in the off-road performance of wheeled and tracked ground vehicles. The roughness of a terrain has typically been quantified by the U.S. Army as the root-mean-square elevation deviation (RMS) of the terrain profile. Although RMS is an important input into many mobility calculations, it is not scale invariant, making it difficult to estimate RMS from low resolution terrain profiles. Fractal parameters are another measure of roughness that are scale invariant, making them a convenient proxy for RMS. While previous work found an empirical relationship between fractal dimension and RMS, this work will show that, by including the cutoff length, an analytic relationship between fractal properties and RMS can be employed. The relationship has no free parameters and agrees very well with experimental data - thus providing a powerful predictive tool for future analyses and a reliable way to calculate surface roughness from low-resolution terrain data in a way that is scale invariant. In addition, we show that this method applies to both man-made ride courses and natural terrain profiles.
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Keywords: Fractal dimension; RMS; Surface roughness

1. Introduction

Surface roughness can be a limiting factor in the speed of cross-country operations of military vehicles. Typically, drivers will not subject themselves to more than 6 W of absorbed power for an extended time. This places an upper limit on speed for a given terrain roughness, often referred to the "ride-limited speed" or simply the "6 W speed". Past experiments have shown a strong correlation between surface roughness, measured in root-mean-square elevation variation (RMS), and ride-limited speed (Murphy and Ahmad, 1986). This correlation makes RMS a useful metric for predicting cross-country mobility, and this methodology has been incorporated into popular analysis tools such as the NATO Reference Mobility Model (NRMM) (Murphy and Randolph, 1994). Input RMS data for these models is often approximated from terrain profiles measured at discrete intervals. This, however, can create inconsistency because RMS is not scale invariant; measurement of elevation on different length scales will produce differing measurements of RMS. It has become customary in the U.S. Army to calculate RMS from elevation profiles measured at 0.30-m intervals. The primary problem is that terrain data of this resolution is typically not available for many areas of interest.

One way to overcome this shortcoming in quantification of surface roughness is to relate roughness to the fractal properties of the terrain. The work of Durst et al. (2011) showed a strong empirical correlation between fractal

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dimension at a 5-m scale and RMS at a 0.30-m scale. In this work, it will be shown that an analytic relationship exists that can be derived from the definitions of RMS and fractal dimension. The analytic relationship has no free parameters and fits the experimental correlation better than the quadratic regression shown in Durst et al. (2011, 2014). More importantly, this relationship is predictive for terrain data of very low resolution, which we will show holds true for resolutions up to 9.1-m.

2. Method

The goal of this work is to show how the fractal properties of a profile sampled at relatively low resolution (9.1 m) can be used to calculate the RMS of the profile that corresponds to higher resolution sampling (0.30 m). This work builds on an extensive body of research on fractal properties of profiles that provide the mathematical foundation for this analysis. In this section, we provide a short overview of what has been discovered regarding the fractal properties of profiles and show this can be applied to calculate the RMS from low-resolution data.

2.1. RMS for mobility calculations

There are several ways to calculate RMS for surfaces and profiles, and while these methods attempt to measure the same property, they may yield somewhat different results (Gallant et al., 1994; Malinverno, 1990; Moreira et al., 1994). Areal-based mobility models such as the NRMM use RMS as input into mobility calculations (Ahlvin and Haley, 1992). Since these calculations are often used in comparative analyses, it is important to define a single method for calculating the mobility parameter RMS_m that is used as input into these models.

The traditional method for calculating the RMS_m of a profile for mobility purposes is defined in Durst et al. (2011). First the profile is detrended to remove the low-frequency variations. For a profile with height measurements $Z(x)$ sampled at discrete points x_i with a spatial resolution of A, the smoothed heights $\overline{Z(x)}$ are given by

$$\overline{Z(x)} = \frac{\sum_{N=0}^{\infty}[Z(x+Nr)+Z(x-Nr)]e^{\frac{-Nr}{\lambda}}}{2\sum_{N=0}^{\infty}e^{\frac{-Nr}{\lambda}}} \quad (1)$$

Here λ is the smoothing length, which is set to 3.048 m in this work and in Durst et al. (2011). The choices of r and λ are typical for ground vehicle mobility considerations (Mason et al., 1985). Once the profile is detrended, the RMS_m is calculated by

$$RMS_m = \sqrt{\frac{\sum_{i=1}^{m}Z(x_i)^2}{m}} \quad (2)$$

2.2. Calculating fractal parameters of profiles

The first important item to note is that the fractal properties of a profile are not fully described by the fractal dimension alone; it is also necessary to calculate the crossover length of the fractal to fully describe its fractal properties (Brown, 1987; Vázquez et al., 2007). While the fractal dimension correlates well to the roughness of a surface *at a given scale*, the crossover length quantifies how the fractal dimension changes with scale. Since the overall goal of this work is to relate the fractal parameters measured at one scale to the RMS values at higher resolutions, it is critical to include the crossover length in the analysis.

A second important point regarding the calculation of fractal parameters is that fractals can generally be either self-similar or self-affine. In simple terms, the distinction is that self-similar fractals retain their fractal dimension under uniform scaling, while self-affine fractals do not. Previous work has shown that topographic profiles are self-affine, not self-similar (Wilson, 2000). This is an important distinction because not all methods for calculating fractal dimension are applicable to self-affine profiles. Specifically, it has been shown that the divider method yields anomalously low results for the fractal dimension of self-affine profiles (Brown, 1987). Previous work on correlating the fractal dimension of profiles of ride-courses failed to take this distinction into account, resulting in extremely low (<1.0001) fractal dimensions (Durst et al., 2011, 2014).

Another important distinction when calculating the fractal dimensions of profiles is to note that there are two general methods for calculating fractal dimensions. The first type are geometric, such as the divider method, roughness-length method. The second type are stochastic methods such as the PSD method. Previous research has shown that these two methods may yield incongruous results for the fractal dimension (Carr and Benzer, 1991), and therefore mixing these two methods, as was done in Durst et al. (2011, 2014), may not yield consistent results.

Several studies have been performed (Liang et al., 2012; Gallant et al., 1994; Carr and Benzer, 1991), and the roughness-length method has been shown to be the most accurate, consistent method for calculating the fractal parameters of self-affine terrain profiles. In this work, we will show how the roughness-length method can be used to extract fractal parameters from sparsely-sampled terrain profiles.

2.3. The roughness-length method

The RMS of a terrain profile can be calculated by dividing the terrain into windows of length h and averaging the root mean square elevation deviation in each window. The number of windows of length h is given by N_h, and the number of points in each window is given by m_h. The RMS is then given by Vázquez et al. (2007)

$$RMS(h) = \frac{1}{N_h}\sum_{u=1}^{N_h}\left[\frac{1}{m_h}\sum_{i\in h}[Z(x_i)-\overline{Z}_h]^2\right]^{1/2} \quad (3)$$