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The in-plane elastic properties of hierarchical composite cellular materials: Synergy of hierarchy, material heterogeneity and cell topologies at different levels

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ABSTRACT

The hierarchical organization of many biological materials plays a key role in their exceptional mechanical properties. Existing studies investigate how hierarchy affects the mechanical behavior of cellular materials and the vast majority of them assume empty cells. In reality, in numerous natural systems the cells are filled with fluids, fibers or other bulk materials to better resist external stimuli. Inspired by the highly efficiency of nature, this paper investigates the effects of adding hierarchy into a composite cellular material. Initially, the analytical expressions for the effective elastic moduli derived in the case of self-similarity reveal the system isotropy as for the not filled configuration. Then, from parametric analysis emerges a strong influence of the microstructure on the overall properties. We discovered that adding hierarchical levels to a filled cellular material can lead to a higher material specific stiffness only if the filler is stiffer than a critical value. Thus for classical cellular materials hierarchy is detrimental for the specific stiffness. In spite of this, for composite cellular solids an optimal number of hierarchical levels naturally emerges. In addition, numerical homogenization validates the analytical approach. Finally, the example of a hierarchical composite cellular material having different levels with different cell topologies is also considered. The present analysis provides an insight into the role of structural hierarchy on the in-plane elastic properties of composite cellular materials, as well as some possible ways to improve low-weight cellular structures by mixing different materials and varying the cell topology.

1. Introduction

It is well known that nature has developed a large number of ingenious solutions that served as a source of inspiration for scientists and engineers (Fratzl and Weinkamer, 2007; Gibson et al., 2010).

In the literature, many works discuss this aspect: among others, the pioneering textbook by Thompson (1992) or, more recently, by Mattheck and Kubler (1995), where the authors extract engineering principles from the structure of trees.

Nowadays, terms like *biomimetics* or *bioinspiration* (Sanchez et al., 2005; Vincent et al., 2006; Fratzl, 2007) are commonly used to describe the new approach in chemistry, material science and engineering. That is, researchers study biological systems to find some useful principles to create and/or improve new materials and simplify many of our day-to-day functions.

Indeed, lessons learned from nature solved a variety of technical challenges in material science (Jeronimidis and Atkins, 1995), architecture (Kemp, 2004), aerodynamics and mechanical engineering (Milwich et al., 2006). For example, most are familiar with the Velcro, inspired by the way plant burrs stuck to animal fur (Cohen, 2005; Jenkins, 2012), the high performance swimsuits, modeled on the structure of shark skin to reduce drag in water (Bixler and Bhushan, 2012) or the super adhesive fabrics that mimic the gecko foot configuration (Shah and Sitti, 2004).

Differently from the engineer, nature has a relatively limited number of structural elements to choose, polymers, composites of polymers and ceramic particles (Fratzl and Weinkamer, 2007). Materials that certainly are not associated with strength, toughness, stiffness or durability. However, even with these restrictions, nature developed a wide range of systems with distinctive functions

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and remarkable mechanical properties that often surpass those of their components by orders of magnitude (Gibson, 2012), as trees, skeletons, shells.

Even though it is still unknown how nature succeeded in doing this, some authors provided a number of possible strategies. Fratzl (2007), for instance, suggests the two paradigms of *growth* and *functional adaptation*, that lead to the complex hierarchical architecture of natural materials. In particular, one advantage of hierarchical structuring is the multi-functionality. That is to say, a specific property, such as fracture toughness, can be tuned at different levels, independently of others properties, and adapted to the local needs (Pan, 2014; Gao, 2010). In other words, the exceptional mechanical behavior of biological systems is due to the functional adaptation of the structure at all levels of hierarchy (Fratzl and Weinkamer, 2007).

In line with these theories, many studies and experimental observations on different natural materials, gecko foot, nacre shell, Armadillo armor, show that hierarchy is the nature's key of success (Chen and Pugno, 2013).

In a system, hierarchy is reflected by several characteristics (Pan, 2014). The first one, multiscality, is the coexistence of several structural levels with gradual transition in length scales ranging from nano to macro scale. The second, heterogeneity, is the presence of different properties at different levels. Also, a variety of designs are possible by changing type and configuration of the constituents (Barthelat and Mirkhalaf, 2013) and, generally, the overall properties rarely reflect those of the constituents. The final characteristics is anisotropy. As a consequence, many mathematical laws and material sciences' principles, that assume isotropy and homogeneity, must be carefully applied to hierarchical systems. General introductions on hierarchical biological materials include the recently published review articles (Fratzl and Weinkamer, 2007; Gibson, 2012; Chen and Pugno, 2013; Pan, 2014; Wegst et al., 2015).

Various authors have extensively studied structural hierarchy. Among them, Lakes (1993) analyzes the hierarchical configuration of some natural materials, as fibrous composites and cellular solids, and of the man-made Eiffel Tower. It emerges that some desirable properties, like stress attenuation, superplasticity and increased toughness, are due to hierarchy. Other authors, like Chen and Pugno (2012); Pugno and Chen (2011); Haghpanah et al. (2014); Ajdari et al. (2012); Fan et al. (2008); Taylor et al. (2011) develop numerical and theoretical models, force or energy based, to understand the role of hierarchy on the in-plane mechanical behavior of cellular solids. In particular, Chen and Pugno (2012) and Haghpanah et al. (2014) focus on the elastic buckling while Pugno and Chen (2011); Ajdari et al. (2012); Fan et al. (2008) obtain analytical expressions for the macroscopic elastic moduli. In addition, Bosia et al. (2012) considers different hierarchical architectures of fiber bundles and, through multiscale calculations, proposes an analytical method to evaluate how hierarchy can affect the structural strength. Specifically, the study shows that, in the case of different types of fibers, the increase in the number of hierarchical levels leads to an improvement in the material strength. In the context of hierarchical materials with a self-similar microstructure, namely when the geometry is similar from one scale to another, several attempts have been made to model their mechanical behavior. Oshmyan et al. (2001) presents a finite element-based technique to evaluate the effective elastic moduli and scaling properties of two-dimensional materials containing self-similar multiscale voids/rigid inclusions whose distribution closely resemble the Serpinski-like carpet. The investigation suggests that increasing the levels of hierarchy provides an increase in the coefficient of anisotropy, leading to a mechanical behavior close to that of unidimensional materials. It also emerges that the scaling laws defining the transition between the properties belonging to different length of scale are power equations whose exponents are function of the inclusions/voids' dimension. A similar result is theoretically obtained in Dyskin (2005), that investigates self-similar media with different types of inhomogeneities and stress concentrators, such as pores, cracks, rigid inclusions. The proposed technique uses the concepts of the differential self-consistent method (Salganik, 1973) where it is assumed that equally-sized inhomogeneities does not interact directly. The interacting ones have different length of scale. The material is also represented as a sequence of homogenized continua of increasing scale, obtained in the average sense. An attempt to numerically solve boundary value problems for self-similar domains structured on a large number of scales is proposed in Soare and Picu (2007). The authors, in particular, present a finite element procedure that employs modified shape functions to capture the complexity of the geometry at no additional computational cost. An extension of the concepts of classical Fracture Mechanics to cracks propagating in a self-similar regime is discussed in Borodich (1999). The analysis, in particular, focuses on scaling laws of fracture energy in brittle and quasi-brittle materials. It emerges that, independently of the material, the fracture energy is an exponential function with the exponent only related to the considered length of scale. Same considerations apply in the case of multiple cracks creating a self-similar pattern.

Inspired by the complex hierarchical organization of natural materials, the present paper focuses on the effects of adding hierarchy into a two-dimensional composite cellular material (Ongaro et al., 2016): namely, a cellular structure having the cells filled by a generic elastic material and a hierarchical architecture. In addition, the study investigates how hierarchy affect the macroscopic in-plane elastic moduli and whether it is possible to improve the specific stiffness by structural hierarchy, material mixing and varying cell topologies at different levels. Finally, as in Ongaro et al. (2016), the Euler-Bernoulli beam on Winkler foundation elements model the microstructure at all levels.

A brief overview is of order. Initially, Section 2 focuses on a composite cellular material with a honeycomb microstructure and n levels of hierarchy. The assumption that the length of scale of the sub-structure is fine enough to be negligible with respect to the super-structure (Lakes, 1993) leads to the elastic constants in the continuum description. Then, Sections 3 and 4 present the comparison between the analytical and numerical approach, as well as the results of the parametric analysis to investigate the influence of the geometrical and mechanical microstructure parameters on the macroscopic properties. In particular, the analysis reveals that adding hierarchical levels to a cellular material can provide a higher material specific stiffness only if the filler is stiffer than a critical value. An optimal number of hierarchical levels also emerges. Conversely, for hollow cellular materials hierarchy is detrimental for the specific stiffness. To the authors' best knowledge, this is the first time such results are reported.

2. The hierarchical composite cellular material: analytical model

2.1. Elastic constants

A hierarchical material contains structural elements which themselves have structure (Lakes, 1993). Also, the hierarchical order of the material, n, can be defined as the number of levels of scale with recognized structure (Lakes, 1993). This paper deals with the in-plane analysis of a hierarchical composite cellular material having n levels of hierarchy. That is to say, a material with n hierarchical levels, a honeycomb-like architecture and the cells filled at each level (Fig. 1). As in Ongaro et al. (2016), a sequence of Euler-Bernoulli beams on Winkler foundation forming a periodic array of hexagonal cells simulates the underlying configuration at all levels. In accordance with Lakes (1993), the length of scale of