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Two-fluid smoothed particle hydrodynamics simulation of submerged granular column collapse

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ABSTRACT

In this paper, a two-fluid smoothed particle hydrodynamics (SPH) model, based on the mixture theory, is employed to investigate the complex interactions between the solid particles and the ambient water during the process of submerged granular column collapse. From the simulation, two regimes of the collapse, one being quick and the other being slow, are identified and the reasons of formation are analyzed. It is found that, a large internal friction angle of the granular phase, representing large drag force between solid particles, helps form the slow regime. Small hydraulic conductivity, representing large inter-phase drag force, also retards the collapse dramatically. Good agreements between our numerical results and other researchers' numerical and experimental results are observed, which demonstrates the capability of the proposed two-fluid SPH approach in dealing with saturated water–soil mixture flows.

1. Introduction

Submerged soil column collapse is a typical water–soil interaction problem and a model for many underwater natural and hazardous processes, such as debris flows [1], landslides [2], submarine avalanches [3], to name but a few. Due to its harmful impacts to the safety of the underwater structures or the geomorphology changing of waterways, submerged granular column collapse has long been a research concern for geophysicists, hydrologists and underwater engineers. There are a large amount of experimental and numerical works dealing with the dry granular flows (such as sand, glass beads, etc.); among them, the so-called $\mu(I)$-rheology [4,5] has recently emerged as a major step toward consistently describing the granular materials. Lagree et al. [6] implemented the $\mu(I)$-rheology in a Navier–Stokes solver and simulated the unsteady 2D collapse of granular columns over a wide range of aspect ratios. Ionescu et al. [7] proposed a $\mu(I)$-rheological numerical model for dry granular column collapse over inclined planes.

Compared to dry soil column collapse, however, the process of submerged soil column collapse has not yet been well understood, due to the complex interactions between water and solid particles. For better understanding of the challenges and the recent developments on this topic, readers may refer to Rondon et al. [8], Meruane et al. [9,10], Savage et al. [11] and the references therein.

During the collapse process, soil undergoes large deformation which is difficult to treat using grid-based methods. Problems that involve history-dependent constitutive models, like large deformation plasticity, are expressed more naturally in a Lagrangian computational frame. For this reason, meshfree Lagrangian particle methods for soil mechanics seem to be increasing in popularity recently.

Bui et al. [12] conducted the numerical simulation of the waterjet–soil interactions using the smoothed particle hydrodynamics (SPH) method which is a fully Lagrangian and meshfree method. They proposed a two-phase model, in which the water is regarded as a Newtonian fluid and the soil as an elastic-perfectly plastic material. Interactions between water and soil were modeled by the Darcy's law and pore water pressure. With the aid of this novel numerical method, some interesting phenomena in water–soil interactions have been revealed. Bui et al. [13] and Bui and Fukagawa [14] proposed an incremental plasticity model to describe the large deformation behavior of soil. Recently, authors of this paper (see [15]) developed an SPH approach for large deformation analysis with hypoplastic constitutive model. Dunatunga and Kamrin [16] developed a material point method (MPM) to simulate dry granular flows with $\mu(I)$ inertial rheology. In Zhang et al. [17], a

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version of the particle finite element method (PFEM) was proposed to analyze the large deformation of granular flow problems.

More recently, we proposed a two-fluid SPH mixture model to analyze the waterjet–soil interaction problem [18–20]. In this model, each constituent of the mixture satisfies its own conservation equations of mass and momentum. Unlike Bui et al. [12], volume fractions of both constituents are taken as field variables which must be determined together with the other fundamental variables, such as pressure and velocities. With this mixture model, it is possible to investigate the temporal and spatial evolutions of the volume fractions of both constituents.

In this paper, the proposed two-fluid SPH mixture model is employed to analyze the water–soil interactions during submerged granular column collapse. Two regimes of the collapse, one is quick and the other is slow, could be observed from the simulations. Key factors influencing the formation of different regimes, namely the internal friction angle and the hydraulic conductivity of the soil, are investigated, which helps reveal the mechanisms of soil failure in the presence of an ambient fluid.

2. Mathematical formulation

2.1. Water–soil mixture model

The role of the ambient fluid can be investigated through a two-phase continuum mixture theory (e.g. [9,18,21]). Mixture theory assumes that, at any time, every point in space is occupied simultaneously by one particle of each constituent. With this assumption, we can define partial densities ρ_η, partial velocities $\boldsymbol{v}_\eta$, and partial stresses $\boldsymbol{\sigma}_\eta$ for each constituent. Here $\eta = l, s$ for fluid and solid, respectively. Each constituent must satisfy individual balance laws for the conservation of mass

$$\frac{\mathrm{D}^\eta \rho_\eta}{\mathrm{Dt}} = -\rho_\eta \nabla \cdot \boldsymbol{v}_\eta, \tag{1}$$

and momentum

$$\rho_\eta \frac{\mathrm{D}^\eta \boldsymbol{v}_\eta}{\mathrm{Dt}} = \nabla \cdot \boldsymbol{\sigma}_\eta + \rho_\eta \boldsymbol{g} + \boldsymbol{f}_\eta, \tag{2}$$

where $\boldsymbol{g}$ the gravitational acceleration, and $\mathrm{D}^\eta(\cdot)/\mathrm{Dt}$ the material time derivative along the path of particles of η phase. The interaction force $\boldsymbol{f}_\eta$ is the force exerted on phase η by the other constituent. By definition, the sum over the two constituents is equal to zero, i.e. $\boldsymbol{f}_l + \boldsymbol{f}_s = 0$.

In mixture theory we should specify how partial variables are related to their physical, or intrinsic, counterparts. In standard mixture theory, the partial and intrinsic velocity fields are identical, while the densities are related by volume fractions, i.e.

$$\rho_\eta = \phi_\eta \tilde{\rho}_\eta, \quad \boldsymbol{v}_\eta = \tilde{\boldsymbol{v}}_\eta, \tag{3}$$

where variables with a tilde denote intrinsic variables, and ϕ_η is the volume fraction of phase η, satisfying $\phi_l + \phi_s = 1$ for a saturated liquid-solid mixture.

For the soil stress, we assume that

$$\boldsymbol{\sigma}_s = \phi_s \tilde{\boldsymbol{\sigma}}_s, \quad \boldsymbol{\sigma}_l = -p\boldsymbol{I} + \phi_l \tilde{\boldsymbol{\tau}}_l, \tag{4}$$

where $\tilde{\boldsymbol{\sigma}}_s$ is the intrinsic stress of the soil, p the pore water pressure, and $\tilde{\boldsymbol{\tau}}_l$ the intrinsic deviatoric stress tensor of the water. The interaction force $\boldsymbol{f}_s$ (i.e. $-\boldsymbol{f}_l$) is assumed to be in the form

$$\boldsymbol{f}_s = -\phi_s \nabla p + C_d(\boldsymbol{v}_l - \boldsymbol{v}_s). \tag{5}$$

Here the second term on the right-hand side is simply an interphase resistance term, with C_d being the drag coefficient. The first term can be identified as a buoyancy force, e.g. the surface pressure exerted across the surface of the solids because of the surrounding fluid. The buoyancy force, combining with $-\nabla p$ in the momentum balance equation of water, leaves $-\phi_l \nabla p$. This ensures, as in Darcy's law, that the percolation process is driven by intrinsic (pore) rather than partial pressure gradients (e.g. [22]).

In the current study, the so-called weakly compressible SPH (WCSPH) method is employed to investigate the fluid dynamics. In WCSPH, the pressure is given by the equation of state of water, in such a manner that the fluctuation of the water density is less than 1% which represents a very weak compressibility. For the SPH method, mass conservation is always assured, because the mass of each particle is fixed and the number of particles remains unchanged during the simulation.

The constitutive relationship and equation of state for water can be seen in Wang et al. [18]. Here we only give a brief description on the constitutive relationship of the soil and the interaction model between water and soil.

2.2. Constitutive laws of soil and interaction forces

In this study, the intrinsic density $\tilde{\rho}_s$ of the solid particles is assumed as constant. Consider the soil as an elastic-perfectly plastic material with a Drucker-Prager yield criterion

$$F(I_1, J_2) = \sqrt{J_2} + \alpha_\theta I_1 - k_c, \tag{6}$$

where I_1 is the first invariant of the total stress tensor $\tilde{\sigma}_s^{\alpha\beta}$, and J_2 is the second invariant of the deviatoric stress tensor $\tilde{\tau}_s^{\alpha\beta}$. In Eq. (6), α_θ and k_c are constants which can be related to the cohesion c and the friction angle θ of the Mohr-Coulomb failure criterion by matching the two models [23, p. 149]. In this paper, cohesion is considered as zero, thus $k_c = 0$. For plane strain problem, α_θ is determined by

$$\alpha_\theta = \frac{\tan\theta}{\sqrt{9 + 12\tan^2\theta}}. \tag{7}$$

In computational plasticity theory, it is assumed that the total strain in a body can be decomposed into an elastic part and a plastic part. The elastic part of the strain can be computed from a linear elastic constitutive law, e.g. the Hooke's law. To model the plastic part, however, we need a flow rule which states how the plastic deformation takes place once the stress threshold has been reached. With this decomposition, the total strain rate tensor $\dot{\varepsilon}_s^{\alpha\beta}$ can be written as

$$\dot{\varepsilon}_s^{\alpha\beta} = \frac{\dot{\tilde{\tau}}_s^{\alpha\beta}}{2G} + \frac{1-2\nu}{3E}\dot{\tilde{\sigma}}_s^{\gamma\gamma}\delta^{\alpha\beta} + \dot{\lambda}\frac{\partial H}{\partial \tilde{\sigma}_s^{\alpha\beta}}, \tag{8}$$

where α, β are free indices and γ is a dummy index; $\delta^{\alpha\beta}$ is Kronecker's delta, $\delta^{\alpha\beta} = 1$ if $\alpha = \beta$ and $\delta^{\alpha\beta} = 0$ if $\alpha \neq \beta$; $\tilde{\tau}_s^{\alpha\beta} = \tilde{\sigma}_s^{\alpha\beta} - \frac{1}{3}\tilde{\sigma}_s^{\gamma\gamma}\delta^{\alpha\beta}$ is the deviatoric part of the total stress tensor $\tilde{\sigma}_s^{\alpha\beta}$; G is the shear modulus and ν the Poisson's ratio; $\dot{\lambda}$ is the rate of change of the so-called plastic multiplier λ dependent on the state of stress and load history and H is the plastic potential function.

Rearranging (8), the general stress–strain relationship for an elastic-perfectly plastic material can be derived as

$$\dot{\tilde{\sigma}}_s^{\alpha\beta} = 2G\dot{e}_s^{\alpha\beta} + K\dot{\varepsilon}_s^{\gamma\gamma}\delta^{\alpha\beta} - \dot{\lambda}\left[2G\frac{\partial H}{\partial \tilde{\sigma}^{\alpha\beta}} + \left(K - \frac{2G}{3}\right)\frac{\partial H}{\partial \tilde{\sigma}^{\gamma\gamma}}\delta^{\alpha\beta}\right], \tag{9}$$

where $\dot{e}_s^{\alpha\beta}$ is the deviatoric part of the strain rate tensor $\dot{\varepsilon}_s^{\alpha\beta}$, K the bulk modulus. $\dot{\lambda}$ can be calculated by using the consistency condition which states that the new stress state after loading still satisfies the yield criterion (6), i.e.

$$\mathrm{d}F = \frac{\partial F}{\partial \sigma^{\alpha\beta}}\mathrm{d}\sigma^{\alpha\beta} = \frac{\partial F}{\partial \sigma^{\alpha\beta}}\dot{\sigma}^{\alpha\beta}\mathrm{d}t = 0. \tag{10}$$

In this paper, two types of flow rule are implemented in the simulation. The first one is the so-called associated flow rule, where