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Research paper

Stiffness and mass optimization of parallel kinematic machine

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ABSTRACT

It has long been a challenge to carry out the optimal design of parallel kinematic machine (PKM) simultaneously considering stiffness and mass performances. This paper proposes the stiffness and mass optimization of PKM by settling performance indices, constraint conditions based on parameter uncertainty and cooperative equilibrium among performances. Firstly, instantaneous energy-based stiffness indices and mass in motion are defined as objectives. Instead of computationally expensive numerical analysis, analytical mapping models between objectives and parameters are investigated to improve optimization efficiency. Then, considering the effects of parameter uncertainty resulted from manufacturing errors during construction, constraint conditions are formulated by probabilistic method. Based on particle swarm optimization (PSO), a multi-objective optimization is implemented. A group of solutions are obtained to flag as Pareto frontier that reflects the competitive features between stiffness and mass performances. A cooperative equilibrium searching method is proposed to find out the final solution. Finally, this optimization approach is exemplified and validated by a five degree-of-freedom (DoF) PKM. Although its mass increases 17.17%, the stiffness is nearly 3 times better than before optimization.

1. Introduction

Parallel kinematic machine (PKM) consists of moving platform, fixed base and at least two kinematic chains [1–4]. It can be designed as compact module and applied to form parallel or hybrid machine tool by integrating with articulated heads or long guideways. PKM has desirable performances in accuracy and rigidity as traditional machines, and in workspace/footprint ratio, dynamic response and flexibility as articulated serial robots [5,6]. It has been recognized as promising solution for machine tools [7,8], telescope positioning [9] and food packaging [10] etc. Although PKMs have found the market in many applications, they are not as successful as expected. Most existing PKMs are expensive devices that provide worse performances than conventional machines. Further investigation is necessary for making PKM more attractive to industries. Among the main issues to be addressed, the optimal design problem is crucial [11].

The purpose of optimal design aims at enhancing performance indices by adjusting dimensional or sectional parameters of PKMs. These parameters should satisfy certain constraints, such as limitation of workspace or maximum errors. Then the parameter adjustment is implemented by optimization algorithm [12]. This approach is called performance optimization of PKMs. In the optimization process, two basic and important performances are usually involved, i.e. stiffness and mass. The former is directly related to machining deformation and accuracy, while the latter affects dynamic performance.

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In the past few decades, optimization of PKMs towards high stiffness has been intensively reported. For instance, Kang [13] defined stiffness metric by using weighted sum of stiffness along different directions, and carried out manipulability and stiffness optimal design of a two degree-of-freedom (DoF) PKM. Bi [14] formulated stiffness matrix of Tripod PKM by Jacobian matrix and then performed its optimization taking weighted objective among stiffness, workspace, dexterity and purity as the optimal function. Shin defined a weighted factor by multiplying the determinant and isotropy of stiffness matrix. He then regarded the weighted sum as single objective function to optimize a redundant PKM [15]. The diagonal elements of stiffness matrix are applied as the objective function to implement stiffness optimal design of 3-DoF and 6-DoF PKMs [12,16].

It is noted from above literature that increasing parameter values benefits the stiffness enhancement. But it will have a negative influence on the dynamic performance of PKMs by enlarging the scale and increasing the weight. Therefore, stiffness and mass performances are competitive and restrain each other. By regarding any of them as the single objective in the optimal design process, the performance optimization becomes a game that two players (stiffness and mass) act according to their own strategies to maximize their individual gains [17]. The resulting solution cannot reach the best interest for both since the players act independently without cooperating with each other [18]. A cooperative play is necessary to deal with optimization of stiffness and mass performances.

There have been two ways to incorporate both stiffness and mass performances into the optimization of PKMs. The first way is regarding natural frequency as objective function. Being formulated by the ratio of stiffness and mass, natural frequency has the merits of unit invariance and clear physical meaning [19,20]. Menon obtained optimal geometric parameters of PKMs with US and UPS kinematic chains through maximizing the first natural frequency in prescribed workspace [21]. Herein, P, U and S denote prismatic, universal and spherical joints. Similarly, Alessandro determined optimal geometric parameters of a 4-DoF PKM by maximizing natural frequency when moving platform is at a fixed pose and within a cube workspace [22]. Although natural frequency is helpful in obtaining a stiffer PKM with lightweight structure, the cooperative game between stiffness and mass performances become a single objective optimization. Ratio of stiffness and mass is to be maximized instead of searching for their best compromise. It might lead to that solutions hidden in concavities cannot be discovered [23].

The other way is to let mass be the objective function and regard stiffness as a constraint condition. Ur-Rehman optimized mass of a 3-DoF PKM by setting ratio of force and deformation, i.e. stiffness value as constraint conditions [24]. Wu obtained geometric parameters of a 3-DoF PKM by taking mass and dexterity as objective functions, and strength as constraint condition. This constraint is calculated by stiffness matrix [25]. In such case, the allowed range of stiffness is pre-defined. The optimization turns into minimizing PKM mass while being subjected to specified stiffness ranges. This constraint transferring method cannot fully reflect the conflicting features between stiffness and mass of PKMs in the performance optimization.

Therefore, it is of vital importance to carry out the multi-objective optimization of PKM considering both stiffness and mass performances without any pre-defined preference. In addition, it is noted that the optimal parameters are directly applied to the construction of physical prototype after the optimization. In this process, parameters in the physical prototype cannot to be exactly the same as the optimized parameters due to the machining and manufacturing errors [26,27]. These parameter uncertainties would lead to the differences in the nominal and actual performances of PKMs. Although manufacturing and assembling errors can be minimized by accuracy improvement strategies, parameter uncertainty cannot be completely eliminated. In order to ensure the actual stiffness and mass performances are still be the same as the optimized results, the effect of parameter uncertainty should be addressed in the stiffness and mass optimization of PKMs.

Besides the consideration of multiple objectives and parameter uncertainty, the selection of optimization algorithm is also of vital importance. Traditional optimization algorithm uses local search procedure to search for the optimum. Convergent stepwise such as gradient, Hessian or linearity is applied. Such method heavily depends on good starting points. Global optima can only be found if the problem has certain convexity properties, or else it may fall into local optima [12]. To this end, global algorithm based on natural evolution has been introduced to the optimization of PKMs, for instance, genetic algorithm (GA) [16,28] and particle swarm optimization (PSO) [29–33]. In particular, PSO is a population-based stochastic optimization technique inspired by the social behavior of bird flocking or fish schooling. It has merits in terms of easy implementation and few parameters to adjust [34], thus is widely applied as optimization algorithm of PKMs. Yun [29] found that the fitness value of PSO is better than the traditional method in the optimal design of a 3-PUPU PKM. Shirazi [30] employed PSO to the optimization of a 6-DoF PKM. He indicated that fast convergence and easy variable constraining can be achieved by PSO. For the multi-objective optimization of a bio-inspired PKM, Zhang [31] used PSO to search for the overall optimal performance. The computing time was only 13.11 min. With similar manner, Wang [32] and Zhang [33] applied standard PSO to the optimal design of Tricept PKM and 3-DoF planar PKM. In their works, the efficiency of PSO was highly recognized. Overall, PSO has no evolutionary operators including crossover and mutation. It can be viewed as the extension and improvement of the working principle of GA. Therefore, PSO is selected as the optimization algorithm in our work.

Having outlined the state-of-art in Section 1, this paper is organized as follows. Formulation of stiffness and mass indices are carried out in Section 2, where the analytical mapping models between performance indices and design variables are established. Section 3 defines the constraint conditions by taking parameter uncertainty into account. The multi-objective optimization method is proposed and a cooperative equilibrium point is defined for obtaining the optimal result in Section 4. Taking a 5-DoF PKM as an example, Section 5 illustrates this stiffness and mass optimization approach before conclusions are drawn in Section 6.