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Research paper

Invariant errors of discrete motion constrained by actual kinematic pairs

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ABSTRACT

Motion of a rigid body is systematically investigated using invariants of line-trajectories, and the invariant errors are proposed for the first time to evaluate accuracy of motion for three actual joints C, H and R. A general spatial motion of a rigid body can be dissolved into the following motion with a reference line, having four DOFs, and the relative motion about and along the reference line, having two additional DOFs. The necessary and sufficient conditions of cylindrical motion, helical motion and rotational motion in both continuous and discrete error forms are respectively derived by invariants of line-trajectories and global invariants with minimal values in differential geometry. For discrete data sets, a novel scheme based on the invariants and their fitting errors obtained by the saddle point programming, is developed to characterize the nominal motion and the error-induced motion. The invariant errors are presented to quantify the accuracy of the discrete error motion of joints C, H and R. Experiment was carried out on a machine tool spindle to demonstrate the advantages of the proposed invariants-based error evaluation scheme. The scheme provides a new method for quantifying accuracy of motion and improving performances of machine tools and robots.

1. Introduction

In a mechanism, a kinematic pair, such as the C, H, R and P pairs, consists of a moving link and a base link. For an ideal kinematic pair, the moving link moves in an ideal manner relative to the base link [1]. In a machine, the actual kinematic pair cannot constrain the moving link ideally when machining errors of joint interfaces and component deformations are taken into consideration. The actual motion of the moving link, termed as error motion in this paper, deviates from the ideal motion. Usually the deviations are small in magnitudes in comparing with the nominal motion.

Accuracy of an actual kinematic pair is the degree of closeness between the actual motion and the ideal motion. The problem is how to compare the actual motion with the ideal one for a kinematic pair. In current practices, evaluations are conducted by measuring the actual trajectories of some characteristic points/lines/surfaces of the moving link, and comparing them with the ideal loci. Previous studies included accuracy of spindles [2] and slide-ways [3] in machine tools. Regardless of the measuring principles and devices used, the point-trajectories and line-trajectories are measured and stored in a discrete data format for error motion [4,5]. Since different points/lines of the moving link trace different trajectories in the base frame, there arise some questions. First of all, how the characteristic points/lines should be chosen? Secondly, for the

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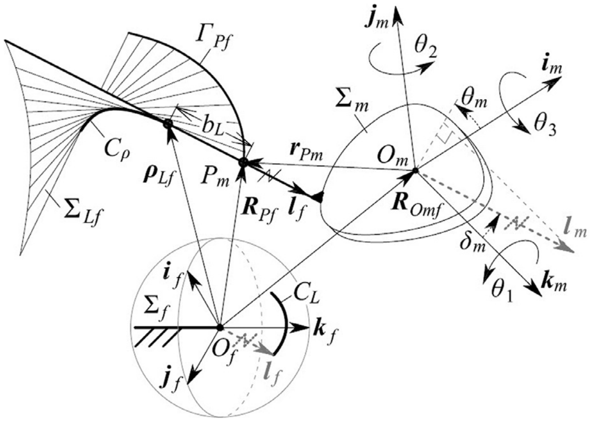

Fig. 1. Spatial motion of a rigid body and a line-trajectory.

chosen points/lines, how the actual error motion should be described using the discrete data? In kinematics, we are interested in knowing whether invariants of the error motion, independent of the point-trajectories and line-trajectories, can be defined. In this paper, we will answer these questions and develop some comprehensive models on basis of the kinematic geometry and the saddle point programming for evaluating error motion.

Invariants of loci have been used to describe rigid body kinematics. Approaches available in the literature include the finite separation positions by Roth [6], the curvature of line-trajectories by McCarthy [7-9], the kinematic constraint by Ge [10,11], the point-line by Ting [12,13], the line geometry by Pottmann [14], the differential geometry by Wang [1,15,16] and other researchers [17–21]. In general, the error motion of small scale is superimposed onto the ideal motion dictated by the type of a joint. The discrete trajectories closely resemble the ideal shape globally; this means the global geometrical properties of the discrete trajectories should be used to describe and evaluate the error motion. However, systematic studies of error motion of actual joints are not available in the literature.

In this paper, the error motion of a link in both continuous and discrete forms is studied using the Frenet frame in differential geometry [1]. Based on the geometrical properties of the ruled surfaces, the invariants of line-trajectories and their minimal values are derived to describe joint-specific motion such as the cylindrical motion, helical motion and rotational motion, and the accuracy of an actual kinematic pair. The work presented in this paper helps to provide a new theoretical basis for evaluating the accuracy of actual kinematic pairs.

2. Description of motion

Spatial motion of a rigid body may be described in various classical ways. In differential geometry, a set of invariants is judiciously introduced to eliminate the dependence of a formulation on the choice of different coordinate systems. They are more convenient in analyzing the geometrical properties of point- and line-trajectories [1].

2.1. Displacements of points and lines

Fig. 1 shows a moving rigid body Σ_m with a body-fixed moving frame $\{O_m;\ \boldsymbol{i}_m, \boldsymbol{j}_m, \boldsymbol{k}_m\}$ can be represented by six coordinates: $(x_{Omf}, y_{Omf}, z_{Omf})$, defining translations of origin O_m, and $(\theta_1, \theta_2, \theta_3)$, defining rotations about O_m. For a prescribed motion, all six parameters are functions of time *t*.

The trajectory of a point in Σ_m, noted by P_m with coordinates (x_{Pm}, y_{Pm}, z_{Pm}) in the moving frame, is a 3D curve Γ_{Pf} in the space-fixed frame $\{O_f; \boldsymbol{i}_f, \boldsymbol{j}_f, \boldsymbol{k}_f\}$. The equation of the curve can be written in the vector form as

$$\boldsymbol{\Gamma}_{Pf} : \boldsymbol{R}_{Pf} = \boldsymbol{R}_{Omf} + [\mathbf{M}_{mf}]\mathbf{r}_{Pm} \tag{1}$$

where $\boldsymbol{R}_{Pf}$ is the position vector of P_m and $\boldsymbol{R}_{Omf} = [x_{Omf}, y_{Omf}, z_{Omf}]^T$ is the position vector of origin O_m in the fixed frame; $\boldsymbol{r}_{Pm} = [x_{Pm}, y_{Pm}, z_{Pm}]^T$ is the position vector of P_m in the moving frame. The rotational matrix $[\mathbf{M}_{mf}]$ for the Euler angles $(\theta_1, \theta_2, \theta_3)$ in Fig. 1 is [22]

$$[\mathbf{M}_{mf}] = \begin{bmatrix} c\theta_1 c\theta_2 & c\theta_1 s\theta_2 s\theta_3 - s\theta_1 c\theta_3 & c\theta_1 s\theta_2 c\theta_3 + s\theta_1 s\theta_3 \\ s\theta_1 c\theta_2 & s\theta_1 s\theta_2 s\theta_3 + c\theta_1 c\theta_3 & s\theta_1 s\theta_2 c\theta_3 - c\theta_1 s\theta_3 \\ -s\theta_2 & c\theta_2 s\theta_3 & c\theta_2 c\theta_3 \end{bmatrix} \tag{2}$$

where s stands for sine and c for cosine.