Mechanism and Machine Theory 119 (2018) 106–118

Contents lists available at ScienceDirect

Mechanism and Machine Theory

journal homepage: www.elsevier.com/locate/mechmachtheory

Research paper

Effect of initial curvature in uniform flexures on position accuracy

M. Verotti

Department of Mechanical and Aerospace Engineering, Sapienza University of Rome, Via Eudossiana 18, 00184 Rome, Italy

ARTICLE INFO

Article history:
Received 22 June 2017
Revised 29 August 2017
Accepted 30 August 2017
Available online 12 September 2017

Keywords:
Compliant mechanisms
Pseudo-rigid-body model
MEMS
Flexures
Center of rotation
Position accuracy

ABSTRACT

Position accuracy is a prerequisite for compliant mechanisms, especially in micro-scale applications. Generally, this feature depends on the flexures ability to imitate the revolute joints of the pseudo-rigid body model. In case of flexible elements, the center of the relative rotation varies its position during the deflections, affecting the position accuracy. In this paper, the role played by the initial curvature is investigated, in case of uniform primitive flexures. Analytical expressions are derived to evaluate the shift of the rotation axis with respect to the flexure centroid. An example shows the performance of four flexures with different curvatures, evaluated by comparing the rotation axis shift and the position error.

1. Introduction

Compliant mechanisms are being increasingly used because they offer some advantages with respect to their rigid-body counterparts, such as reduced friction and backlash, no need for lubrication, simplified manufacture, fabricability at macro, meso and micro scale, and compatibility with MEMS-based technologies.

Compliant mechanisms have been applied, for example, in the fields of micro-manipulation [1–5], micro-positioning [6–10], precision manufacturing [11,12]. Applications in these areas require high position accuracy, therefore precision of analytical models represents a significant challenge in compliant mechanisms design [13,14].

Generally, the synthesis of a compliant mechanism can be performed by means of freedom and constraint topologies [15–17], topology optimization [18–20], building blocks [21–23], or rigid-body replacement [24–26]. The rigid-body replacement method consists in obtaining a compliant mechanism by replacing the revolute joints of a rigid-body mechanism, or pseudo-rigid body model (PRBM), with flexible elements. In case of straight-axis primitive flexures, the position of the flexible element with respect to the revolute joint can be determined according to different criteria. If the length of the flexure is small if compared to the subsequent rigid links, its midpoint can be placed on the position of the revolute joint. If the length of the flexible beam is comparable to the rigid elements, its position with respect to the revolute joint can be determined by parametric approximation of the beam's deflection path, by calculating the characteristic radius factor [27,28]. Another possible criteria consists in positioning the center of rotation of the flexure on the revolute joint [29].

The position accuracy of a compliant mechanism depends on the mechanical characteristics of the flexure and on its position with respect to the connecting links. In other words, the motion accuracy of the compliant mechanism depends on the flexure ability to imitate the behaviour of the revolute joint. Whilst the latter guarantees a fixed position for the center

E-mail address: matteo.verotti@uniroma1.it

http://dx.doi.org/10.1016/j.mechmachtheory.2017.08.021
0094-114X/

of the relative rotation of the connecting links, the deformation of the flexure depends on the applied loads. Therefore, the center of rotation varies its position during motion. This condition is referred to as axis shift or parasitic motion [30,31].

To improve the performance of compliant systems in terms of range of motion and precision of rotation, but also in terms of stress concentration or off-axis stiffness to axial stiffness ratio [32], many investigations have been focused on the development of complex flexures. These flexures combine more flexible elements or involve contact systems [33–37]. In fact, the variation of the position of the center of rotation occurs especially in primitive flexures, such as straight- or curved-axis beams, undergoing large deflections. In case of end-moment loads, the center or rotation moves along the axis of symmetry of the flexure, and its distance from the axis centroid increases for increasing loads [29].

Recent investigations focused on the adoption and on the modeling of circular flexures in compliant mechanisms. Wang et al. used the PRB to model corrugated beams [38], and Han et al. designed a quadstable monolithic mechanism based on curved elements [39]. Edwards et al. proposed a PRB model for curved beams spinned at both ends [40], whereas Venkiteswaran and Su developed a PRBM for circular beams subjected to combined loads [41].

With respect to straight-axis beams, curved flexures offer more flexibility in the design phase, introducing a new parameter, that is the radius of curvature. This parameter can be set in order to optimize the geometry of the compliant mechanism, for example to minimize the distance between two subsequent rigid links or to overcome geometrical constraints. However, this parameter has an effect on the position of the center of rotation of the flexure. As a consequence, it affects also the position accuracy of the compliant mechanism.

This paper focuses on the role played by the initial curvature of uniform flexures on their ability to mimic the rotational accuracy of revolute joints. The analysis is performed considering constant cross-section beams subjected to end-moment loads. Analytical expressions are derived to determine the position of the center of rotation with respect to the beam axis centroid. Various axis curvatures are considered, including the straight-axis case.

2. Position of the center of rotation of the free-end section

Considering a constant-curvature beam with axis length equal to l, the Euler-Bernoulli beam equation can be written as

$$\frac{d\theta}{ds} = \frac{\mu}{l}, \tag{1}$$

where s is the arc-length coordinate system, $d\theta/ds$ is the rate of change of angular deflection along the beam, and

$$\mu = \mu_0 + \tilde{\mu} \tag{2}$$

is a non-dimensional term equal to the sum of

$$\mu_0 = \frac{l}{r}, \tag{3}$$

where r is the radius of curvature, and

$$\tilde{\mu} = \frac{M}{EI} l, \tag{4}$$

where M is the bending moment and EI is the bending stiffness.

With reference to Fig. 1, the application of an external moment to the flexure free-end, represented by $\tilde{\mu}$, determines a transition from the initial configuration, $\mathcal{C}_{\mu_0}$, to the deformed one, $\mathcal{C}_{\mu}$. The orientation of the free-end section varies from the angle θ_E to the angle θ_F, and its centroid moves from point E to point F. The position of the pole of the displacement, or center of rotation, C_r, is given by [42]:

$$\frac{x_{C_r}}{l} = \frac{1}{2l}\left(x_E + x_F - (y_F - y_E)\cot\frac{\theta_F - \theta_E}{2} \right), \tag{5}$$

$$\frac{y_{C_r}}{l} = \frac{1}{2l}\left(y_E + y_F + (x_F - x_E)\cot\frac{\theta_F - \theta_E}{2} \right), \tag{6}$$

where the Cartesian coordinates are normalized with respect to the beam axis length. As demonstrated in Ref. [29], a convenient representation of the C_r coordinates can be obtained by introducing the reference frame $\mathcal{R}_O\{O, x, y, z\}$, whose origin is positioned on the centroid of the clamped section, and whose $y-$axis is parallel to the arc chord. With respect to $\mathcal{R}_O$, the position of the free-end section in the neutral and deformed configurations are given by

$$\boldsymbol{p}_E^0 = \left[0,\ \frac{\sin\alpha}{\alpha} \right]^T, \tag{7}$$

and

$$\boldsymbol{p}_F^0 = \left[\frac{1}{\mu}(\cos(\alpha - \mu) - \cos\alpha),\ \frac{1}{\mu}(\sin\alpha - \sin(\alpha - \mu)) \right]^T, \tag{8}$$