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Original paper

Theoretical analysis on effects of grain size variation

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ABSTRACT

Grinding wheels consist of abrasive grains, bonding materials and porous, which are specified by five factors; type of grain, size of grain, bonding material, bonding strength and grain concentration or volume fraction of grain. The grain size is represented by the mean diameter/radius of grains. However, the abrasives in a grinding wheel are randomly scraggly in size and shape. There is no particular aspect to regulate the variation in grain size. This paper addresses, via a theoretical analysis on the distribution of grain size, the average volume of a grain, the number of grains contained in a specific volume of the wheel, the number and protrusion distribution of grains exposed in a wheel working surface and the fraction of effective grain, when the grain size varies. The effect on the resultant finished surface roughness is also discussed.

1. Introduction

Grinding wheels consist of three elements of abrasive grains, bonding materials and porous, which are specified by five factors; type of grain, size of grain, bonding material, bonding strength and grain concentration or volume fraction of grain. Fig. 1 shows the configurations of grinding wheels in typical surface grinding dynamics; (a) rotational in-feed face grinding and (b) conventional surface grinding. Focusing on the wheel working surface, the grinding performance of a wheel is then dominated by the grain size, the cutting edge geometry, the number and protrusion height distribution of cutting edges, because they significantly influence the grinding force, surface roughness, wheel service life and etc. [1–3]. So far, many studies have been made to understand the effects of cutting edge geometry [4,5], size [5,6], protrusion height [7,8], density [8], kinematical cutting path [9,10] and self-sharpening characteristics [2].

However, there is no particular aspect to regulate the variation in grain size. In most studies mentioned in the above references, the grain size is represented by the mean diameter/radius of grains. The average volume of a grain, the number of grains contained in a specific volume of the wheel or other parameters respect to grain size, volume and density are simply calculated by use of the mean diameter/radius of grains. Very few researchers have paid attention to the effect of variation in grain size although it is mentioned in some papers like [5,11].

Via a theoretical analysis on the distribution of grain size in grinding wheels, this paper studies the effect of standard deviation of grain size on the average volume of a grain, the number of grains contained in a specific volume of the wheel, the number and protrusion distribution of grains exposed on a wheel working surface and the fraction of effective grain. The effect on the resultant finished surface roughness is also discussed.

2. Grinding wheel model description and number of grains contained in a specific volume of wheel

Unlike conventional abrasives, diamond grains are hardly broken or fractured to form multiple cutting edges during truing, dressing and grinding operations. Therefore, most grains exposed on the wheel working surface remain in their original crystal forms and each individual grain always represents a single cutting edge [1,12]. This is especially true for fine grit diamond wheels. Illustrated in Fig. 2 is one piece of diamond wheel segment with the dimension of $L \times W \times H$. To build up an analytical model of the wheel working surface, the following assumptions have been made;

Grains are approximated into spheres with the radius r_g.

r_g varies in accordance with a normal distribution (r_0, σ), where r_0 is the mean grain radius, and σ is the standard deviation of the grain radius.

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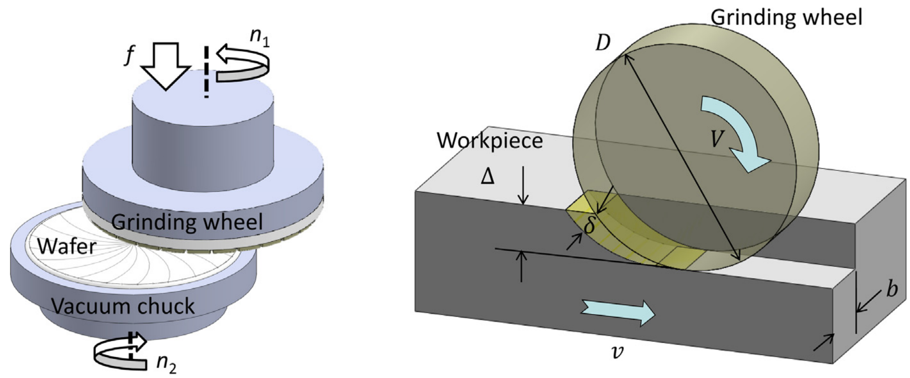

(a) Rotational in-feed face grinding (b) Conventional surface grinding

Fig. 1. Surface grinding dynamics.

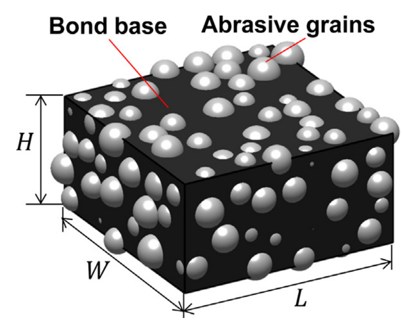

Fig. 2. Modeled wheel segment.

It should be noted although the theoretical analysis in this paper is made on a diamond wheel for simplicity, the results obtained in the following section is also valid to conventional grinding wheels.

Fig. 3 shows an example of the actual radius variation of diamond grains, which are used in the production of SD1000 wheels. The grain radius variation is well matched with a truncated normal distribution as expressed in Eq. (1).

$$f\left(r_g\right) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{\left(r_g - r_0\right)^2}{2\sigma^2}\right] \tag{1}$$

where $r_0 = 6.0\mu$m, $\sigma = 1.5\mu$m. The grains with radius $r_g \left\langle r_0 - m\sigma \text{ or } r_g \right\rangle r_0 + m\sigma$ are then truncated by filtration. In other words, the grains used in wheel manufacturing satisfy $r_0 - m\sigma < r_g < r_0 + m\sigma$. The multiplier $m = 1$ is favored by most wheel manufacturers. Therefore, the grain size distribution is governed by two factors; the standard deviation σ and the multiplier m. Discussed in the subsequent sections are the effects of σ and m.

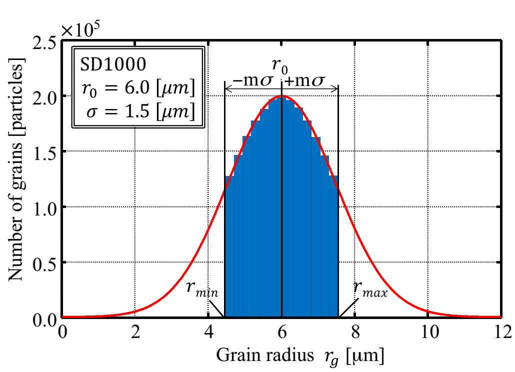

Fig. 3. Grain size in a normal distribution.

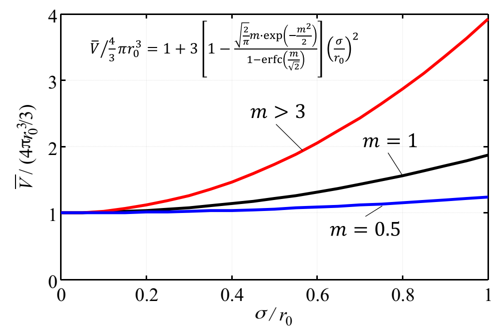

Fig. 4. Average volume of grains.

When the radius r_g varies within the range of $r_0 - m\sigma < r_g < r_0 + m\sigma$, the average volume $\bar{V}$ of a single sphere is mathematically expressed as;

$$\bar{V} = \frac{4}{3}\pi\frac{\int_{r_0+m\sigma}^{r_0-m\sigma} r_g^3 f\left(r_g\right)dr_g}{\int_{r_0+m\sigma}^{r_0-m\sigma} f\left(r_g\right)dr_g} = \frac{4}{3}\pi r_0^3$$

$$+4\pi\left[1 - \frac{\sqrt{\frac{2}{\pi}}m\cdot\exp\left(-\frac{m^2}{2}\right)}{1 - erfc\left(\frac{m}{\sqrt{2}}\right)}\right]r_0\sigma^2 \tag{2}$$

where, erfc () is referred to the *complementary error function*.

Eq. (2) clearly states that the average volume $\bar{V}$ of varied r_g is larger than $4\pi r_0^3/3$, the volume calculated by using of the mean radius r_0. Fig. 4 shows the normalized relationship between $\bar{V}$ and σ. The $\bar{V}$ increases with the square of σ, and m accelerates the increasing rate in $\bar{V}$. Compared with $4\pi r_0^3/3$, the actual $\bar{V}$ could be several folds larger. This fact results in less number of grain contained in the wheel than expected. Eq. (3) gives the number of grain N contained in a specific volume of wheel where the concentration or volume fraction V_g of abrasives in the wheel is given.

$$N = \frac{V_g}{\bar{V}} = \frac{3V_g}{4\pi r_0^3}\cdot\frac{1}{1 + 3\left[1 - \frac{\sqrt{\frac{2}{\pi}}m\cdot\exp\left(-\frac{m^2}{2}\right)}{1-erfc\left(\frac{m}{\sqrt{2}}\right)}\right]}\left(\frac{\sigma}{r_0}\right) \tag{3}$$

Fig. 5 shows the variation of the normalized N as a function of the normalized σ. Obviously, larger $\bar{V}$ leads to less number of grains. It is worthy to emphasize again that the actual number of grain with varied r_g could be a substantially small fraction of number calculated by using of the mean radius r_0, if the abrasives are broadly distributed in size. For a wheel with the same grain volume

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