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Design and stiffness analysis of a compliant spherical chain with three degrees of freedom

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ABSTRACT

This paper introduces and investigates a compliant spherical 3R open chain that is obtained by the in-series connection of three identical circularly-curved beam flexures with coincident centers of curvature and mutually orthogonal axes of maximum rotational compliance. The considered open chain is intended to be used directly as a spherical mechanism in pointing devices or as a complex spherical flexure for the development of spatial parallel manipulators. The compliance matrix of the proposed chain is first determined via an analytical procedure. After finite element validation, the obtained equations are used in a parametric study to assess the influence of circularly-curved beam flexure geometric parameters on the overall stiffness performances of the considered compliant spherical 3R open chain. In addition, comparison with an equivalent compliant spherical chain employing straight beam flexures is reported to highlight the added benefits of using circularly-curved beam flexures in terms of reduced parasitic motions.

1. Introduction

Compliant mechanisms (CMs) are a special kind of articulated systems in which motion, force or energy are transferred or transformed through the deflection of flexible members (hereafter briefly referred to as "flexures" or "flexural hinges") [1]. Thanks to the absence (or reduced use) of traditional kinematic pairs, which are instead based on mating surfaces, CMs are almost not affected by wear, friction and backlash, and only require minimal maintenance with no need of lubrication. Due to their hinge-less nature, CMs can be manufactured in a single piece (for instance via laser or water jet cutting, electrical discharge machining or additive manufacturing), thereby reducing number of parts, assembly needs and, thus, manufacturing costs. With the above-mentioned features, CMs are ideal to work in vacuum, contamination-free, wet or dirty environments and in devices requiring resistance to shocks and silent operation. Common applications of CMs span high-precision manufacturing [2,3], minimally invasive surgery [4,5] and micro-electromechanical systems (MEMS) [6,7].

For what concerns background literature, several studies have been devoted to the design, the characterization and the comparative evaluation of different flexure geometries and CMs formed therewith (see e.g. [8–11]). In particular, most of these devices have been specifically conceived for the generation of planar motions only, out-of-plane displacements being regarded as *parasitic effects* to be minimized as much as possible [12]. On the other hand, despite the huge potentialities, exploitation and study of CMs specifically conceived for spatial motions have been much more rare (see e.g. [13–21]). Within this scenario, the development of Spherical CMs (SCMs) has recently attracted the attention of several researchers. SCMs are an important class of flexure-based spatial CMs in which all points of the end-link are ideally constrained to move on concentric spherical surfaces that are fixed with respect to the grounded link. In particular, the in-series ensemble of two or three compliant revolute (R) joints (of either planar notch, planar leaf spring or straight torsion beam type) with orthogonal and intersecting axes has been proposed in [22–25] to conceive compliant spherical 2R or 3R serial chains to be used as compliant universal or spherical joints for the development of Cardan's [26] and Double-Hooke's couplings [25] and of spatial parallel manipulators [14–21]. In these applications, the use of compliant spherical 2R or 3R serial chains in place of the axial-symmetric notch primitive flexure is usually preferred owing to the more limited ranges of motions and larger stress concentrations of this latter. The connection of four, five, six or eight bars with an equal number of compliant revolute

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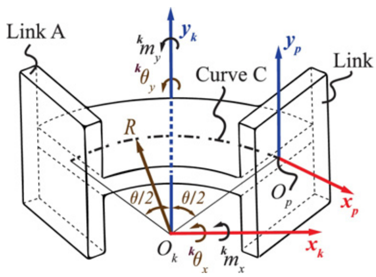

Fig. 1. Circularly-curved beam flexure (CCBF).

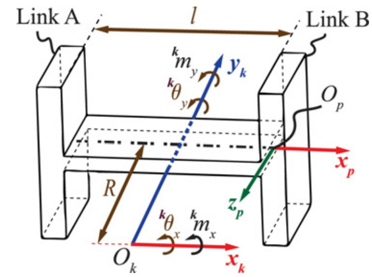

Fig. 3. Straight beam flexure (SBF).

joints (of either straight crease or lamina emergent torsional type) with intersecting axes has been considered in [27–29] for the development of 4R, 5R, 6R or 8R closed single-loop lamina-emergent SCMs, as well as combinations thereof (including the six bar Watt's and Stephenson's linkages), to be used in origami-inspired foldable systems such as pop-up books, industrial packaging and deployable devices. Planar notch and straight torsion beam flexures have been used in [24] to develop an actuated miniature 3-CRU (C and U denoting cylindrical and universal joints respectively) spherical parallel CM for the orientation of parts and tools in space. The in-parallel connection of three symmetrically placed spherical 3R serial chains employing either lamina emergent straight torsion beam or notch flexures has been proposed in [30,31] for the development of 3-(3R) spherical parallel CMs with flat initial state to be used in compact pointing devices such as in MEMS beam-steering mirrors or medical instruments.

In all the above-mentioned studies, the considered SCMs have been obtained by employing compliant revolute flexures specifically conceived for planar motion applications. In contrast to this, Circularly-Curved Beam Flexures (CCBFs) (Fig. 1) with constant cross-section and featuring lower rotational rigidity along the radial direction have been proposed in [32–34] for the development of SCMs with improved spherical motion capabilities.

In this context, this paper investigates the use of CCBFs (Fig. 1) for the development of compliant spherical 3R serial chains to be used as SCMs or as spherical complex flexure components for spatial CMs with either serial or parallel architectures. As depicted in Fig. 2, the considered spherical chains are obtained by the in-series connection of three identical CCBFs that are arranged in space so as to share the same center of curvature (point O_0) and have mutually orthogonal axes (x_0, y_0, z_0) of maximum rotational compliance. Analytical results validated via finite element analysis are first provided to show that these spherical 3R serial chains made with CCBFs (see Fig. 2) always exhibit lower parasitic motions than those of equivalent SCMs, featuring identical primary rotational compliance, but made with Straight Beam Flexures (SBFs, see Figs. 3 and 4).

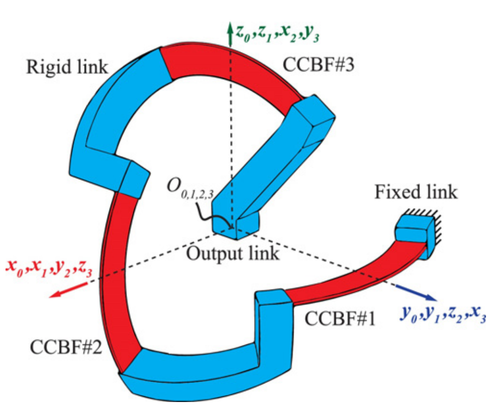

Fig. 2. CCBF-based compliant spherical 3R open chain.

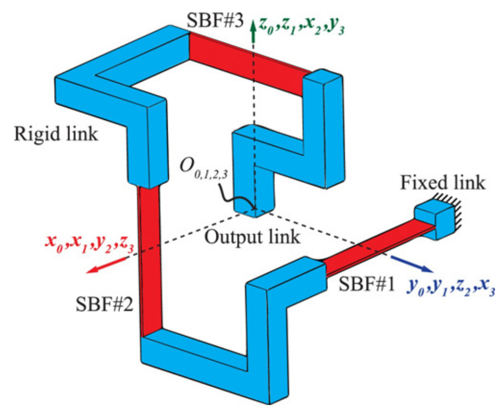

Fig. 4. SBF-based compliant spherical 3R open chain.

Then, design graphs are provided which can be used to select CCBFs geometric parameters for known primary rotational compliance and parasitic motions requirements of the considered SCM.

2. Closed-form compliance equations for compliant spherical 3R open chains

2.1. Basic notation and background theory

In general, recalling the basic notation previously proposed in [35], flexural hinges are conceived in order to allow one or more *primary rotations* along desired reference directions when subjected to *primary loads* acting along the same directions. The ratio between any primary displacement and its related primary load can be defined as *primary compliance*. For instance, both the CCBF and the SBF depicted in Figs. 1 and 3 (which connect a fixed rigid link *A* to a movable rigid link *B*) can act as revolute joints by generating a primary rotation ${}^k\boldsymbol{\theta} = [0 \quad {}^k\theta_y \quad 0]^T$, under the action of a primary load ${}^k\mathbf{m} = [0 \quad {}^km_y \quad 0]^T$ due to the application of a force or moment at some point of link B (e.g. point O_p).[1] In this case, the hinge primary compliance is the ratio between ${}^k\theta_y$ and ${}^km_y$. Similarly, flexible members that are specifically conceived to provide multiple Degrees of Freedom (DoF) will be characterized by one primary compliance factor related to each desired DoF. Parasitic effects (or *secondary displacements* [12]) may occur due to the presence of *secondary loads* or to the presence of *axis drift* (i.e. hinge undesired motion even in absence of secondary loads). Ratios between secondary displacements and loads are referred to as *secondary compliances*. A given CM topology is said to provide a *selectively compliant* behavior if it maintains large primary compliance factors but small secondary compliances.

[1] In the following, the left superscript of a vector or a matrix will denote the coordinate frame in which its components are expressed.