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A new adaptive sequential sampling method to construct surrogate models for efficient reliability analysis

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ABSTRACT

Surrogate models are often used to alleviate the computational burden for structural systems with expensively time-consuming simulations. In this paper, a new adaptive surrogate model based efficient reliability method is proposed to address the issues that many existing adaptive sequential sampling reliability methods are limited to the Kriging models and Krging model-based Monte Carlo simulation (MCS) reliability methods produce random results even without considering the uncertainty from initial samples. Three learning functions are developed for selecting the most suitable training sample points at each iteration, and the learning functions ψ_σ and ψ_m are generally suggested because they were found to perform a bit better in most of the cases. Furthermore, most of the newly selected training sample points are ensured to reside far away from existing sample points and reside as close to the limit-state functions as possible. Two stopping criterions are given to terminate the proposed adaptive sequential sampling algorithm. The main advantages of the proposed method are that it not only provides an efficient manner for structural reliability analysis with multiple failure modes to produce a determined result under without considering the uncertainty from initial samples, but also can be used, in principle, in any existing surrogate models. The accuracy and efficiency as well as applicability of the proposed method are demonstrated using three numerical examples.

1. Introduction

Uncertainty widely exists in practical engineering and it usually comes from the inherent randomness (variation) associated with a physical system or the environment [1,2]. The performance and reliability of a structural system are affected by unavoidable uncertainties. Therefore, it is necessary to consider various uncertainties for a structural system to ensure high reliability [3].

Major concerns in structural reliability analysis are the estimation of probability of failure and reliability sensitivity under uncertainties. Based on linear and quadratic approximations of the performance function at the most probable point (MPP), the first order reliability method (FORM) and the second order reliability method (SORM) are widely used for structural reliability problems due to their good balance between accuracy and efficiency [4–6]. However, for a highly non-linear performance function with a large number of inputs, the results obtained by FORM/SORM are not accurate enough due to the non-normal to normal transformations and multiple MPPs as well as applying only first/second order terms to approximate the original performance functions [7]. New reliability methods have been developed, such as the mean value first order saddlepoint approximation [8], univariate approximation [9], subset simulation [10,11], evidence theory [12], improved weighted average simulation [13], and importance sampling based methods [14,15], to improve the accuracy and/or to address the shortcomings of FORM/SORM methods. Despite advances in the field of component reliability, reliability analysis for structural systems with multiple failure modes remains a challenge due to the complicated features and intersections as well as highly nonlinear correlations for the multiple failure modes. Therefore, bounds of system probability of failure are provided by many reported reliability methods, instead of its exact value [16].

Finite element (FE) simulation is widely used for many structural systems with expensively time-consuming implicit performance functions. The cost of a large number of repeated FE simulations is extremely expensive. High skills are generally required for structural reliability analysis involving FE simulations. In order to improve compu-

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tational efficiency and to avoid a large number of expensive FE simulations, surrogate-models are widely used to alleviate the computational burden. Several surrogate models are available for structural reliability problems, and the widely used one is the second-order polynomial regression model due to its simplicity and high computational efficiency. Relevant research works are usually called response surface methods (RSMs) which can be found in many publications (e.g. Refs. [17–20]). Despite the usefulness of RSMs, sometimes they cannot approximate relationships between inputs and outputs accurately for the highly non-linear performance functions. Therefore, reliability analysis using RSMs should be with caution for the highly non-linear performance functions. In order to improve the accuracy, some other surrogate models (e.g. neural networks (NNs), Kriging models, support vector machines (SVMs), radial basis functions (RBFs) and splines) have been used. Generally, a surrogate model should be constructed to approximate performance function (the original function) across the uncertainty space of interest, and it can be used as a full replacement of the original function. Two ways [21], called "one-shot" and sequential sampling design, can be used to construct an accurate surrogate model. In the "one-shot" experimental design, input-output training sample points are collected in advance, and a surrogate model (e.g. NNs, Kriging models, SVMs, RBFs and splines) is constructed based on the collected training sample points. The number of training sample points is increased until the accuracy of the constructed model is satisfied. The "one-shot" experimental design methods are more popular because they are easy to use and understand [22]. However, the total number of sample points needs to be determined in advance which is very difficult for many black-box problems. In sequential sampling design, sample points are selected using an iterative method. The information of each iteration is used for selecting additional new training samples. The central question for sequential sampling design methods is where the selected new training sample points should be located at each iteration [23].

Adaptive sequential sampling methods using Kriging models (also called Gaussian process (GP) models), rather than other surrogate models (e.g. NNs, SVMs, RBFs and splines), have been reported in recent years for structural reliability problems with time-consuming implicit performance functions because Kriging models provide both the mean prediction and prediction variance for an un-sampled point [24]. Bichon et al. [25] proposed an efficient global reliability analysis (GERA) method for structural systems with implicit performance functions and the expected feasibility function was developed. Echard et al. [26] proposed a learning function (i.e. U function) to select new training samples, and an active learning reliability method combining Kriging and crude MCS (AK-MCS) was developed. In order to further improve the computational efficiency of the AK-MCS, Zhu and Du [27]proposed a new reliability analysis method based on the AK-MCS that all statistical information from Kriging models (i.e. mean prediction, prediction variance and correlations) was considered simultaneously. Hu and Mahadevan [28]used global sensitivity analysis to select training samples and a new adaptive Kriging model based reliability method was proposed. More recently, wen et al. [29] proposed an adaptive sampling regions strategy to select samples, and a sequential Kriging based reliability analysis method is presented. Sun et al. [30]proposed a new learning function named least improvement function (LIF) for selecting new training samples to construct an adaptive Kriging surrogate model for efficient reliability analysis. Gaspar et al. [31] used trust region based method to search the MPP, and an adaptive Kriging surrogate model with active refinement was proposed. It is worth noting that adaptive surrogate models based on Kriging were widely used for reliability problems. The main rationale for using Kriging models is that the prediction variance for an un-sampled point can be used to guide the selection of subsequent training sample points [24]. However, the prediction variance estimations for other surrogate models (e.g. NNs, SVMs, RBFs and splines) are difficult to obtain. Therefore, existing adaptive sequential sampling approaches based on the Kriging models cannot be used for other surrogate models (e.g. NNs, SVMs, RBFs and splines). We also note that the Kriging model-based reliability methods may not be efficient for multiple failure modes because Kriging models are generally available for systems with multi-inputs and one output. Furthermore, Krging model-based MCS reliability methods produce random results even without considering the uncertainty from initial samples. In this paper, neural network is selected to construct a surrogate model due to its applicability to higher dimensional problems, the ability of approximation for arbitrary functions, and high level of accuracy in local and global approximations [32]. Another advantage of neural network is that it is suitable for systems with multi-inputs and multi-outputs simultaneously which is very useful for structural systems with multiple failure modes and large dimensions.

To the best of our knowledge, several surrogate models (e.g. Kriging models, NNs, SVMs, splines and RBFs) are available for approximation, each of which with their own merits [33]. In order to address the issues that many existing adaptive sequential sampling reliability methods are limited to the Kriging models and Krging model-based MCS reliability methods produce random results even without considering the uncertainty from initial sample points, a new adaptive sequential sampling method is proposed in this paper to construct surrogate models for efficient reliability analysis. The proposed method can be applied to structural systems with multiple failure modes and expensively time-consuming implicit performance functions. The main advantages of the proposed method are that it is not only effective for structural reliability analysis without producing a random result, but also can be used, in principle, in any existing surrogate models (e.g. Kriging models, NNs, SVMs, RBFs and splines).

The rest of the paper is organized as follows: an efficient adaptive sequential sampling method for structural reliability is proposed in Sections 2; three numerical examples are used to demonstrate the accuracy and efficiency as well as applicability of the proposed method in Section 3; conclusions are given in Section 4 to close the paper.

2. The proposed method for efficient reliability analysis

High skills and a large number of repeated FE simulations are main disadvantages of existing reliability methods for structural systems with expensively time-consuming implicit performance functions and multiple failure modes. In order to address the issue that many current adaptive sequential sampling methods are limited to the Kriging models, a novel adaptive sequential sampling method is proposed to construct a back-propagation (BP) neural network for efficient reliability analysis. Three learning functions are developed to select the most suitable training sample points at each iteration.

The following three main steps are used for structural reliability analysis in the proposed method. The details of each step are given in the following subsections.

(1) Construct initial surrogate models using BP neural network;
(2) Use learning functions to select the most suitable training sample points at each iteration;
(3) Construct the final surrogate model and calculate the system probability of failure.

2.1. The proposed adaptive sequential sampling algorithm

Generally, a structural system with multiple failure modes can be expressed as

$$\begin{aligned} Z_1 &= g_1(\mathbf{X}) \\ Z_2 &= g_2(\mathbf{X}) \\ &\vdots \\ Z_{m_s} &= g_{m_s}(\mathbf{X}) \end{aligned} \tag{1}$$

where $Z_j = g_j(\mathbf{X})$ is the jth performance function, Z_j is the response with $j = 1, 2, \cdots, m_s$. m_s is the number of failure modes, $\mathbf{X} = (X_1, X_2, \cdots, X_n)$ is a vector of random variables. For the jth failure mode $Z_j = g_j(\mathbf{X})$, the safe and failure regions are defined as $Z_j > 0$ and $Z_j < 0$, respectively.