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Letter

Magnetic induction strength on surface of a ferro-medium circular cylinder

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HIGHLIGHTS

- An analytical integral expression of relation between magnetic flux leakage (MFL) and magnetization of a ferro-medium circular cylinder is found.
- Magnetization of material can be predicted by the MFL on surface the circular cylinder.

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ABSTRACT

Based on the Ampere molecular current hypothesis and the Biot–Savart law, a magnetic model on the metal magnetic memory (MMM) testing of a specimen is proposed. Relation between magnetic flux leakage (MFL) and magnetization of a ferro-medium circular cylinder is set up. We can predict magnetization of material according to the MFL on surface of the circular cylinder.

Metal magnetic memory (MMM) method is one of nondestructive tests on metal specimens. Many experiments showed that stresses have some effects on magnetic memory signal on surface of a metal specimen, i.e., magnetic flux leakage (MFL). Wilson et al. [1,2] used a three axis magneto-resistive magnetic field sensor to measure the residual magnetic fields parallel to the applied stress and the material surface and perpendicular to the material surface generated by the magneto-mechanical effect without the application of an external field. Stupakov et al. [3] presented a magnetic investigation of structure degradation of low-carbon steel due to plastic tensile strain. Li et al. [4] gained the magnetic memory signal data in fatigue experiments to 16MnR base metal specimens. Dong et al. [5,6] showed that the normal component of magnetic memory signals depends on the applied stress. Shi et al. [7] showed that normal component of scattering magnetic field of 18CrNiA steel in static tension is linear. Li and Xu [8] measured surface magnetization on 1045 and A3 steel samples uniaxially deformed to differing magnitudes of plastic strain. Experiments of Roskosz and Bieniek [9,10] show that magnetization depends on type of material, strength of magnetic field, magnetic history, strain, and temperature. Leng et al. [11] explored magnetic memory signals of mild steel under uniaxial tensile stresses approaching and exceeding the macroscopic elastic limit and after unloading. A review on this issue in detail can be found in the paper of Wang et al. [12].

However relation between stress and magnetic memory signal on surface of a metal specimen is not very clear theoretically now. One of these problems is the relation between magnetization in material and the MFL on surface of a specimen. The Ampere molecular current hypothesis is a hypothesis on origin of magnetization in modern physics. Based on the Ampere molecular current hypothesis, magnetism in medium is caused by internal molecular current. The MFL on surface of material is caused by many internal molecular currents in the ferro-medium. Therefore the MFL on surface of material should be quantitatively dependent on strength of the molecular currents in the ferro-medium. In this paper, based on the Ampere molecular current hypothesis, a theoretical model is proposed to investigate the relation between magnetization in material and the MFL on surface of a circular cylinder specimen.

This paper presents the Biot–Savart law, the theoretical model of a circular cylinder, numerical results of normal magnetic induction strength on and over surface of a circular cylinder.

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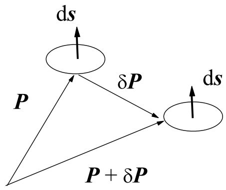

Fig. 1. Solid angle.

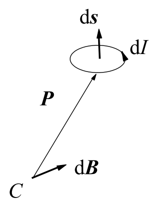

Fig. 2. Magnetic induction strength $\boldsymbol{B}$ by the small element with current circle dI.

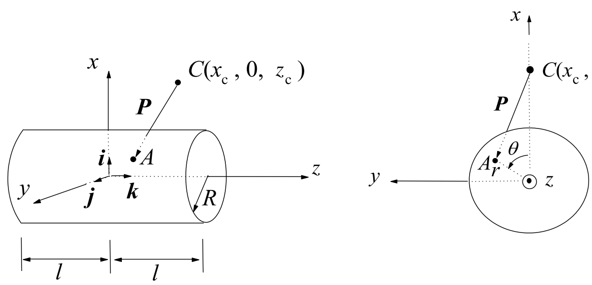

Fig. 3. A circular cylinder of ferro-medium.

A solid angle of an area element d$\boldsymbol{s}$ to the point C in Fig. 1 is defined as

$$\Omega = \frac{\boldsymbol{P}\cdot \mathrm{d}\boldsymbol{s}}{|\boldsymbol{P}|^3}, \tag{1}$$

where $\boldsymbol{P}$ is position vector of the area element d$\boldsymbol{s}$, $|\boldsymbol{P}|$ is length of the vector $\boldsymbol{P}$, and symbol "·" is the scalar product of two vectors. In Eq. (1), we have gradient of the solid angle with respect to $\boldsymbol{P}$

$$\nabla\Omega = \frac{1}{|\boldsymbol{P}|^5}\left[|\boldsymbol{P}|^2\,\mathrm{d}\boldsymbol{s} - 3\left(\boldsymbol{P}\cdot\mathrm{d}\boldsymbol{s}\right)\boldsymbol{P}\right]. \tag{2}$$

In Fig. 2, based on the Biot–Savart law, contribution of the magnetic induction strength of the small element with a current circle dI to the point C is

$$\mathrm{d}\boldsymbol{B} = \frac{\mu\,\mathrm{d}I}{4\pi}\nabla\Omega, \tag{3}$$

where μ denotes magnetic conductivity ratio.

Figure 3 is a circular cylinder of ferro-medium. xyz and $r\theta z$ are the Cartesian coordinates and the circular cylindrical coordinates with the same origin, respectively. $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ are the unit vectors along axis of the Cartesian coordinates xyz. $A\,(r,\theta,z)$ and $C\,(x_c, 0, z_c)$ are an inner point and an outpoint of the circular cylinder, respectively. Vector CA in Fig. 3 is

$$\boldsymbol{P} = (r\cos\theta - x_c)\,\boldsymbol{i} + r\sin\theta\,\boldsymbol{j} + (z - z_c)\,\boldsymbol{k}, \tag{4}$$

where $r \in (0, R)$, $\theta \in (-\pi, \pi)$, $z \in (-l, l)$, $x_c \in (R, \infty)$, and $z_c \in (-\infty, \infty)$.

Based on the Ampere molecular current hypothesis, the magnetization of ferro-medium is caused by many current circles in the ferro-medium.

We suppose that there is a current circle dI around the point A on surface of the differential element as shown in Fig. 4, where

$$\mathrm{d}I = M\mathrm{d}z, \tag{5}$$

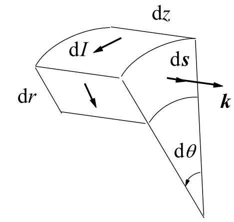

Fig. 4. A current circle dI around point A.

and where M is the magnetization or increase of the magnetization at the point A. For example, increase of the magnetization may be caused by stress or deformation, in elastic or in plastic, in the circular cylinder. The area element d$\boldsymbol{s}$ in Fig. 4 is

$$\mathrm{d}\boldsymbol{s} = r\mathrm{d}r\mathrm{d}\theta\boldsymbol{k}. \tag{6}$$

From Eqs. (4), (6), we have

$$\boldsymbol{P}\cdot\mathrm{d}\boldsymbol{s} = (z - z_c)\,r\mathrm{d}r\mathrm{d}\theta. \tag{7}$$

Substituting Eqs. (4), (6), (7) into Eq. (2), we have the gradient of the solid angle of d$\boldsymbol{s}$ to point C in Fig. 3, i.e.,

$$\begin{aligned}\nabla\Omega = \frac{1}{|\boldsymbol{P}|^5}&\{-3\,(z-z_c)\left[(r\cos\theta - x_c)\,\boldsymbol{i} + r\sin\theta\,\boldsymbol{j}\right] \\ &+ \left[|\boldsymbol{P}|^2 - 3\,(z-z_c)^2\right]\boldsymbol{k}\}r\mathrm{d}r\mathrm{d}\theta.\end{aligned} \tag{8}$$

Substituting Eqs. (5), (8) into Eq. (3), we have the contribution of the magnetic induction strength of the differential element in Fig. 4 to the point C in Fig. 3, i.e.,

$$\begin{aligned}\mathrm{d}\boldsymbol{B} = \frac{\mu M}{4\pi}\frac{r}{|\boldsymbol{P}|^5}&\Big\{-3\,(z-z_c)\left[(r\cos\theta - x_c)\,\boldsymbol{i} + r\sin\theta\,\boldsymbol{j}\right] \\ &+ \left[|\boldsymbol{P}|^2 - 3\,(z-z_c)^2\right]\boldsymbol{k}\Big\}\mathrm{d}r\mathrm{d}\theta\mathrm{d}z.\end{aligned} \tag{9}$$

In Eq. (9), component of the magnetic induction strength normal to surface of the circular cylinder, i.e., component of d$\boldsymbol{B}$ in direction of x, is

$$\mathrm{d}B_x = -\frac{3\mu M}{4\pi}\frac{r(r\cos\theta - x_c)(z - z_c)}{|\boldsymbol{P}|^5}\mathrm{d}r\mathrm{d}\theta\mathrm{d}z. \tag{10}$$

Suppose the magnetization M distributes uniformly in the circular cylinder, by integrating Eq. (10), we have the component of the magnetic induction strength normal to surface of the circular cylinder at the point C in Fig. 3, i.e. the normal component of MFL on and over surface of the circular cylinder

$$B_x\,(x_c, z_c) = \frac{\mu M}{2\pi} f\,(\bar{x}_c, \bar{z}_c)\,, \tag{11}$$

where

$$f\,(\bar{x}_c, \bar{z}_c) = \int_0^{\pi} g\,(\theta, \bar{x}_c, \bar{z}_c)\,\mathrm{d}\theta, \tag{12}$$

and (see Eq. (13) in Box I) where

$$\bar{z}_c = \frac{z_c}{R}, \quad \bar{x}_c = \frac{x_c}{R}, \quad \bar{l} = \frac{l}{R}. \tag{14}$$

Because g in Eq. (13) is an odd function with respect to $\bar{z}_c$, f in Eq. (12) is an odd function with respect to $\bar{z}_c$, i.e.,

$$f\,(\bar{x}_c, -\bar{z}_c) = -f\,(\bar{x}_c, \bar{z}_c)\,. \tag{15}$$

Numerical results of integral (Eq. (12)) are shown in Figs. 5–8.

In Eqs. (11), (12), we know that tangent of $B_x \sim \bar{z}_c$ curve is proportional to the magnetization M of material. For example, tangent in Eq. (11) at $\bar{x}_c = 1$, $\bar{z}_c = 0$ is

$$\left.\frac{\partial B_x}{\partial \bar{z}_c}\right|_{\substack{\bar{x}_c=1\\ \bar{z}_c=0}} = \frac{\mu M}{2\pi}\int_0^{\pi}\left.\frac{\partial g}{\partial \bar{z}_c}\right|_{\substack{\bar{x}_c=1\\ \bar{z}_c=0}}\mathrm{d}\theta, \tag{16}$$