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Finite element models with node-dependent kinematics for the analysis of composite beam structures

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ABSTRACT

This paper presents refined one-dimensional models with node-dependent kinematics. The three-dimensional displacement field is discretized into two domains, namely cross-section domain and axis domain. The mechanical behaviors of the beam can be firstly captured by the cross-section functions then interpolated by the nodal shape functions of the beam element. Such a feature makes it possible to adopt different types of cross-section functions on each element node, obtaining node-dependent kinematic finite element models. Such models can integrate Taylor-based and Lagrange-type nodal kinematics on element level, bridging a less-refined model to a more refined model without using special coupling methods. FE governing equations of node-dependent models are derived by applying the Carrera Unified Formulation. Some numerical cases on metallic and composite beam-like structures are studied to demonstrate the effectiveness of node-dependent models in bridging a locally refined model to a global model when local effects should be accounted for.

1. Introduction

Application of composite materials has attracted significant attention over the past several decades to improve the structural efficiency. However, the anisotropy of multi-layered structures makes it computational costly to capture their responses under external loads.

One of the most important issues in numerical modeling is saving computational costs. A major approach is only using refined higher-order models in regions where sophisticated effects have to be described, while employing less refined models in the rest of the structure. Some noticeable methods have been proposed to couple different models. Slender structures can be approximated with beam models. The most classical beam model is Euler-Bernoulli beam, which applies to isotropic beam-like structures with high slenderness ratio. For stubby beam-like structures the shear effects can be captured with Timoshenko [1] beam model. However, to better capture the behavior of composite laminated beams, more reliable models are needed.

Over the last several decades, many refined beam models have been proposed. To consider the deformation of cross-sections, Vlasov [2] proposed the use of warping functions for beam models, this approach has been applied by Friberg [3], Ambrosini et al. [4] and Mechab et al. [5] to capturing the key phenomenon of cross-sectional warping of thin-walled structures. Kim and Lee [6] recently applied a hybrid model based on Euler-Bernoulli and Vlasov models to the study of thin-walled beam including functionally graded materials. Schardt [7] proposed Generalized Beam Theory (GBT) by expanding the displacement field with reference to the mid-plane of the cross-section thin-walled beam. GBT was also adopted by Davies and Leach [8] and Davies et al. [9], and then further extension to the analysis of composite structures was proposed by Silvestre and Camotim [10]. Berdichevsky [11] proposed the Variation Asymptotic Method (VAM) which uses a characteristic cross-section parameter to construct an asymptotic expansion of the solution, this approach was also adopted by Giavotto et al. [12]. Volovoi et al. [13], Yu et al. [14] and Yu and Hodges [15] further applied VAM to composite beam-like structures.

Carrera [16] and Carrera et al. [17] proposed a new methodology, which is known as Carrera Unified Formulation (CUF), as a new framework to construct 1D and 2D models for the analysis of multilayered composites. For 1D (beam) models, CUF introduces functions $F_\tau(x,z)$ (based either on series expansion or interpolation polynomials) to approximate a cross-section. Numerical accuracy can be improved by increasing the number of expansions in a convenient way as demonstrated by Carrera et al. [18], while

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cumbersome derivation of governing equations can be avoided thanks to the introduction of *Fundamental Nucleus*, FN, which is the core unit of the structural stiffness matrix. Such an advantage leads to a variety of models with variable kinematics, including both 1D models by Carrera et al. [19] and 2D models by Cinefra et al. [20] and Cinefra and Valvano [21].

The above-described refined models improve the numerical accuracy at the expense of increasing the computational costs. For example, CUF-based FE models increase the number of degrees of freedom at each node to better approximate the structural responses. Composite material may undergo to local effects as delamination [22], cracks [23] or local buckling [24], these phenomena require accurate models to be predicted. If refined models are only used in specific regions with sophisticated effects (such as high gradients of stress) to be captured leaving the rest of the structure modeled with lower-order models, a compromise between accuracy and consumption can be reached. The coupling of two computational domains has attracted significant attention, leading to various global-local analysis methods.

To enforce the compatibility of the displacements at the interface of the two domains, Prager [25] used a set of Lagrange multipliers, which was further extended to beam models in the framework of CUF [26]. Aminpour et al. [27], and Ransom [28] employed a spline method to couple two domains with different meshes. Similar approaches in the framework of three-field formulation were also reported by Brezzi and Marini [29]. Blanco et al. [30,31] presented an eXtended Variational Formulation (XVF) to couple non-matching kinematic models based on Lagrange multiplier method, which was also adopted by Wenzel et al. [32].

Fish et al. [33] developed an accelerated multi-grid method to speed up the iterative process when sharing the information between coarse and fine meshes. Fish [34] put forward *s*-version finite element method, which improves the accuracy in the local domain by superimposing additional elements with higher-order hierarchical kinematics on the global model, and continuity of displacement can be guaranteed by imposing homogeneous boundary conditions on the superimposed field. Park et al. [35] proposed a similar method which also refines the local mesh without using transition region nor multi-point constraint. The *s*-version FE method was also used in combination with *h*-version [36] and *p*-version models [29], leading to simultaneous multiple model approaches, as summarized by Reddy and Robbins [37] and Reddy [38].

By introducing an overlapping zone to bridge the two domains, Ben Dhia [39] and Ben Dhia and Rateau [40] suggested Arlequin method to impose compatibility within the overlapping domain with Lagrange multipliers. Such an approach has also been implemented in CUF-based models by Biscani et al. for beam models [41] and plate models [42,43]. Hu et al. [44,45] applied Arlequin method in the linear and non-linear multi-scale analysis of sandwich structures. He et al. [46] adopted Arlequin method to bridge low- and high-order models constructed in the framework of CUF, and Constrained Variational Principle (CVP) was used to derive beam elements for layered structures with independent kinematic description in each layer.

Some special techniques that can be used to mix elements with different mesh refinement or of different types have also been implemented in commonly used commercial software. In Rigid Beam Element (RBEi) and Multi-Point Constraints (MPCs) (such as in NX NASTRAN), the dependent degrees of freedom are expressed as a linear function of the independent degrees of freedoms. Such approaches can be used to connect two sets of incompatible elements in simultaneous analyses. ABAQUS provides so-called "Shell-to-solid coupling" which allows for a transition from 2D modeling to 3D modeling. This method uses a set of internally defined distributing coupling constraints to connect nodes along the edge of a 2D model to a set of nodes on a solid surface. Submodeling is a two-step technique, in which the local model is driven on the boundaries nodes by the displacement field obtained with an aforehand global model. The drawback of such an approach is that the change of stiffness of the local model cannot be updated in the global model. A superelement can be treated as an individual element that is defined by grouping a set of elements, and *condensing* the so-called internal degrees of freedom. Such a technique suits the analysis of large-scale structures and parallel computation. All these approaches adopt special coupling functions on the interfaces between the local and the global model or employ special matrix operation techniques. Meanwhile, at least two sets of separately meshed models are needed.

CUF-type displacement functions make it possible to implement node-dependent kinematic FE models. When it comes to refined 1D (beam) models, cross-section functions defined on different nodes can be integrated into the same 1D element by the nodal Lagrangian shape functions. By the introduction of fundamental nucleus, as has been elucidated in Ref. [47], the governing equation can be derived and expressed in a compact way. Such a methodology permits the possibility of connecting to domains with different kinematics by commonly used nodal shape functions without using any specially designed coupling methods, which reduces the complexity of the numerical methods significantly. Such an approach was firstly presented by Carrera and Zappino [48], then extended to the global-local analysis of laminated composite plates by Zappino et al. [49] as well as [50]. As a simplified case, through-the-thickness variable kinematics was discussed by Dehkordi et al. [51] for sandwich plates, and by Carrera et al. [52] for laminated shells.

In the present work, node-dependent kinematic one-dimensional models are applied to construct global-local FE models, and special attention is paid to the analysis of composite structures where the use of refined models is mandatory to obtain accurate results. The governing equations for beam models with node-dependent kinematics are firstly derived by applying Principal of Virtual Displacement (PVD). Numerical results on thin-walled isotropic beam and multi-layered composite beam, as well as a composite thin-walled beam are reported.

2. Preliminaries

Consider a slender structure as shown in Fig. 1, in which the axial direction is along the y direction, the displacement vector can be expressed as:

$$\boldsymbol{u}^T = \{u_x(x,y,z), u_y(x,y,z), u_z(x,y,z)\} \tag{1}$$

where u_x, u_y and u_z are the three displacement components. The

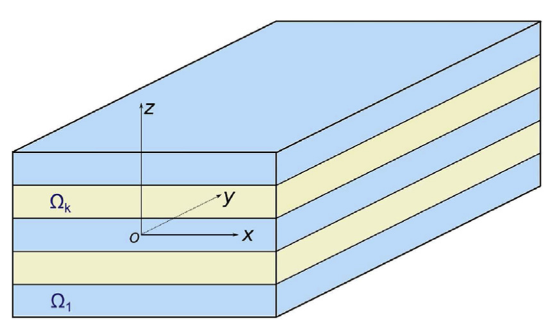

Fig. 1. Reference system of a laminated beam model.