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Determination of the excitation frequencies of laminated orthotropic non-homogeneous conical shells

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ABSTRACT

The excitation frequencies of parametric vibration of laminated non-homogeneous orthotropic conical shells (LNHOCSs) under axial load periodically varying with time, are determined using the classical shell theory (CST). The basic equations are found using the Donnell-Mushtari shell theory and reduce to the Mathieu-Hill type differential equation, in which the instability is examined by the Bolotin method. To validate of current results was made a comparison with the previous studies. The effects of stacking sequences, axial load factors, non-homogeneity, as well as the variation of geometric characteristics on the backward and forward excitation frequencies (BFEFs) of conical shells are studied in detail.

1. Introduction

Layered conical shells are widely used in the aerospace and marine industries due to advantages such as high rigidity to weight and durability, as well as low operating costs. Layered shells can be subjected to dynamic loads under various operating conditions. Thus, the vibration behavior of layered conical shells under dynamic loading is critical for safety and reliability. Among the dynamic problems have been widely studied the free vibrations of homogeneous multilayer conical shells and there are many studies in the literature [1–24].

Modern laminates are inhomogeneous with heterogeneities ranging from a nanoscale to a macroscale [25]. One of the areas of interest is the study of the behavior of the mechanical properties of inhomogeneous layered shells under dynamic periodic loading. Compared to laminated homogeneous shells, the adoption of continuous change of material properties of the layers can provide important benefits. Indeed, the increase in the number of constructive variables extends the possibilities of advanced composite materials, as well as stability and vibration behaviors may be significantly improved. The first basic knowledge on the changes of the material properties is given in the works of Lomakin [26] and Khoroshun et al. [27]. Following these works, numerous studies in this subject have been published in the literature [28–39].

The studies on the parametric vibration of laminated shells are relatively scarce and most of these works are devoted laminated cylindrical shells. One of first study on the solution of the parametric vibration of laminated anisotropic shells is proposed by Goroshko and Emelyanenko [40]. The instability zones of laminated orthotropic cylindrical shell under periodic loads are presented by Argento and Scott [41,42]. The dynamic instability of layered shells under different form of time dependent loads have analyzed by Liao and Cheng [43,44], Lam and Loy [45], and Ng and Lam [46]. A comprehensive bibliography of papers on the parametric vibration of structural elements from 1987 to 2005, are presented by Sahu and Datta [47]. The dynamic instability of layered composite plates and cylindrical shells subjected to uniform and non-uniform axial loads has been studied by Fazilati and Ovesy [48,49]. The dynamic and parametric instability of composite panels is investigated by Dey and Ramachandra [50] and Panda et al. [51], respectively. Dynamic instability analysis for S-FGM plates embedded in Pasternak elastic medium using the modified couple stress theory is studied by Park et al. [52]. Lei et al. [53] presented parametric analysis of frequency of rotating laminated CNT reinforced FG cylindrical panels. Sahmani and Aghdam [54] studied instability of hydrostatic pressurized hybrid FGM exponential shear deformable nanoshells based on the nonlocal continuum elasticity. Li et al. [55]

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investigated nonlocal vibration and stability in parametric resonance of axially moving nanoplate. Akhavan and Ribeiro [56] presented geometrically non-linear periodic forced vibrations of imperfect laminates with curved fibers by the shooting method.

The foregoing brief literature survey reveals that excitation frequencies of parametric vibration of LNHOCS under axial load periodically varying with time have not been investigated to date. This task is undertaken in the current study.

2. Basic relations

The LNHOCS which composed of N layers of equal thickness, as shown in Fig. 1. Terms of contact between any two adjacent layers is absolutely rigid connection that satisfies Kirchhoff-Love hypothesis for the entire shell. The coordinate system $(Or\theta z)$ is located on the mid-surface, in which r, z and θ are axes in the meridional direction, normal to the r axis and in the direction perpendicular to the (Sz) surface, respectively. The orthotropy axes are parallel to the r and θ.

The elasticity moduli of non-homogeneous material of the layer (k+1) are defined as:

$$\left[E_r^{(k+1)}(z_1), E_\theta^{(k+1)}(z_1), G^{(k+1)}(z_1),\ \rho^{(k+1)}(z_1)\right] = \eta_1^{(k+1)}(z_1)\left[E_{0r}^{(k+1)}, E_{0\theta}^{(k+1)}, G_0^{(k+1)}, \rho_0^{(k+1)}\right], z_1 = z/h \tag{1}$$

where $-h/2 + kh/N \le z \le -h/2 + (k+1)h/N, \quad k = 0, 1, \ldots, (N-1)$, $E_{0r}^{(k+1)}$, $E_{0\theta}^{(k+1)}$ and $G_0^{(k+1)}$ are elasticity moduli of homogeneous (H) materials in the layer, $(k+1)$. Additionally, $\eta_1^{(k+1)}(z_1) = 1 + \mu\eta^{(k+1)}(z_1)$, where $\eta^{(k+1)}(z_1)$ denote the variations of the elasticity moduli in the layers and are continuous functions and μ is a variation coefficient of material properties and satisfied the condition $0 \le \mu \le 1$ [30]. The density of H orthotropic materials, $\rho_0^{(k+1)}$, and Poisson's ratios in the layer $(k+1)$, $\nu_{r\theta}^{(k+1)}$ and $\nu_{\theta r}^{(k+1)}$, are assumed to be constant and $\nu_{\theta r}^{(k+1)} E_{0r}^{(k+1)} = \nu_{r\theta}^{(k+1)} E_{0\theta}^{(k+1)}$ [57,58].

Assume that the LNHOCS under the axial load periodically varying with time,

$$n_r^0 = -P(t) = -P_s - P_d\cos(\Lambda t),\quad n_\theta^0 = 0,\quad n_{r\theta}^0 = 0 \tag{2}$$

where n_r^0, n_θ^0 and $n_{r\theta}^0$ are the membrane forces, P_s and P_d are the static axial load and the amplitude of the time dependent periodic axial load, and Λ is the excitation frequency and t is a time variable [59].

The relationships between stresses and strains for a non-homogeneous orthotropic lamina (k+1), in thin conical shells within the Donell-Mushtari theory can be expressed as

$$\begin{bmatrix} \tau_r^{(k+1)} \\ \tau_\theta^{(k+1)} \\ \tau_{r\theta}^{(k+1)} \end{bmatrix} = \begin{bmatrix} E_{11}^{(k+1)} & E_{12}^{(k+1)} & 0 \\ E_{12}^{(k+1)} & E_{22}^{(k+1)} & 0 \\ 0 & 0 & E_{66}^{(k+1)} \end{bmatrix} \times \begin{bmatrix} \varepsilon_r^0 - z\dfrac{\partial^2 w}{\partial r^2} \\ \varepsilon_\theta^0 - z\left(\dfrac{1}{r^2}\dfrac{\partial^2 w}{\partial\theta_1^2} + \dfrac{1}{S}\dfrac{\partial w}{\partial r}\right) \\ \varepsilon_{r\theta}^0 - z\left(\dfrac{1}{r}\dfrac{\partial^2 w}{\partial r\,\partial\theta_1} - \dfrac{1}{r^2}\dfrac{\partial w}{\partial\theta_1}\right) \end{bmatrix} \tag{3}$$

where $\tau_r^{(k+1)}, \tau_\theta^{(k+1)}, \tau_{r\theta}^{(k+1)}$ are the stresses in the layer, $(k+1)$, $\varepsilon_r^0, \varepsilon_\theta^0, \varepsilon_{r\theta}^0$ are the deformations on the reference surface and the quantities $E_{ij}^{(k+1)}$, $(\mathrm{i}, \mathrm{j} = 1, 2, 6)$, are defined as

$$E_{11}^{(k+1)} = \frac{E_{0r}^{(k+1)}\eta_1^{(k+1)}(z_1)}{1 - \nu_{r\theta}^{(k+1)}\nu_{\theta r}^{(k+1)}},\quad E_{12}^{(k+1)} = \nu_{\theta r}^{(k+1)}E_{11}^{(k+1)} = \nu_{r\theta}^{(k+1)}E_{22}^{(k+1)},$$
$$E_{22}^{(k+1)} = \frac{E_{0\theta}^{(k+1)}\eta_1^{(k+1)}(z_1)}{1 - \nu_{r\theta}^{(k+1)}\nu_{\theta r}^{(k+1)}},\quad E_{66}^{(k+1)} = 2G_0^{(k+1)}\eta_1^{(k+1)}(z_1). \tag{4}$$

The force and moment resultants of LNHOCSs are expressed by the following relations:

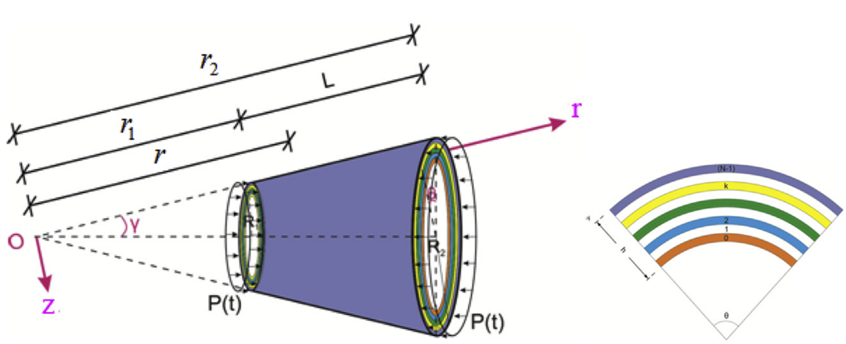
$$[(n_r, n_\theta, n_{r\theta}), (m_r, m_\theta, m_{r\theta})] = \sum_{k=0}^{N-1}\int_{-0.5h+kh/N}^{-0.5h+(k+1)h/N} \times\left[\tau_r^{(k+1)}, \tau_\theta^{(k+1)}, \tau_{r\theta}^{(k+1)}\right][1, z]\mathrm{d}z \tag{5}$$

The Airy stress function, $\Psi(r, \theta, t)$, is introduced by the following relations [58]:

$$(n_r, n_\theta, n_{r\theta}) = \frac{1}{r^2}\left(\frac{\partial^2\Psi}{\partial\theta_1^2} + r\frac{\partial\Psi}{\partial r},\ r^2\frac{\partial^2\Psi}{\partial r^2},\ -r\frac{\partial^2\Psi}{\partial r\partial\theta_1} + \frac{\partial\Psi}{\partial\theta_1}\right) \tag{6}$$

3. Basic equations

The basic equations of LNHOCSs taking into account Eqs. (1) and (2) are expressed as [30,58]:



a) b)

Fig. 1. (a) The LNHOCS under axial load periodically varying with time and **(b)** laminate schema.