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Finite element model for vibration and buckling of functionally graded beams based on the first-order shear deformation theory

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ABSTRACT

This paper presents a finite element model based on the first-order shear deformation theory for free vibration and buckling of functionally graded beams. The present element has five nodes and ten degrees-of-freedom. Material properties vary continuously through the beam thickness according to the power-law form. Governing equations are derived with the aid of Lagrange's equations. Natural frequencies and buckling loads are calculated numerically for different end conditions, power-law indices, and span-to-depth ratios. Accuracy of the present element is demonstrated by comparisons with the available results.

1. Introduction

Functionally graded materials (FGMs) are special composites formed of two or more constituents with a continuous spatial variation. They are usually made of a mixture of ceramics and metals, and can thus resist high-temperature environments while keeping their strength. Therefore, they have been preferred in different applications in aerospace, marine, mechanical, and civil engineering. Increasing demand to FGMs necessitates to well understand the mechanical behavior of such structures.

Compared to functionally graded plates and shells, research on functionally graded beams (FGBs) are relatively less. Bending, buckling and vibration problems of FGBs were solved by different analytical and numerical methods based on various beam theories. Among analytical works, we can mention the followings: Aydogdu and Taskin [1] investigated free vibration of simply-supported FGB based on classical, parabolic and exponential shear deformation theories. Sina et al. [2] developed a novel beam theory different from the classical first-order shear deformation theory to analyze free vibration of FGBs. They assumed the lateral normal stress of the beam is zero. Thai and Vo [3] studied bending and free vibration of FGBs based on various higher-order shear deformation theories. They took into account higher-order variation of transverse shear strain through the depth of the beam with satisfying stress-free boundary conditions. Nguyen et al. [4] developed the first-order shear deformation theory for statics and free vibration of axially loaded FGBs with rectangular cross-section. They derived the improved transverse shear stiffness from the in-plane stress and equilibrium equation, and thus the shear correction factor was obtained analytically. In the foregoing works, researchers used Navier's method to solve governing equations. With the aid of the method of Lagrange multipliers, Şimşek [5] studied free vibration of FGBs considering different higher-order beam theories. He also investigated forced vibration of FGBs under the action of moving loads [6–8]. A higher-order theory with the assumption of hyperbolic distribution of transverse shear stress was proposed by Nguyen et al. [9] for vibration and buckling analyses of FG sandwich beams. Li [10] and Li et al. [11] presented a simple and efficient analytical method for analyzing static and dynamic behaviors of FGBs based on the theory of elasticity. They derived a single fourth-order governing equation, and expressed all physical quantities in terms of the solution of the resulting equation. Li and Batra [12] derived analytical relations between the critical buckling load of a FGM Timoshenko beam and that of the corresponding homogenous Bernoulli-Euler beam subject to axial compressive load. Based on an analogy between FG orthotropic Saint-Venant beams under torsion and inhomogeneous isotropic Kirchhoff plates, Barretta and Luciano [13] proposed an effective solution procedure with no kinematic boundary constraints. Most recently, analytical works related to problems of FG nanobeams [14–18] have been also appeared in the literature.

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Finite element method (FEM) is one of the mostly used numerical method in analyses of structures. Some authors developed different finite element models based on various beam theories in analyzing FGBs. Chakraborty et al. [19] proposed a beam element based on the first-order shear deformation theory to study the thermoelastic behavior of FGBs. They considered the static, free vibration and wave propagation problems to highlight the difference of FGM beam with pure metal or pure ceramic beams. Based on the classical beam theory, Alshorbagy et al. [20] developed a two-noded, six degrees-of-freedom finite element to investigate free vibration of FGBs. Their element can be capable of considering the material graduation in both axial and transversal direction. Kapuria et al. [21] presented a finite element model based on a third-order zig-zag theory for dynamic analysis of layered FGBs. They also gave some experimental results for validation of their proposed theory. Vo et al. [22] developed a finite element model for vibration and buckling of FG sandwich beams based on a refined shear deformation theory. In the formulation, they considered the bending and shear components of transverse displacement as C^1-continuous whereas the axial displacement is C^0-continuous. Most recently, Vo et al. [23,24] presented a two-noded C^1-continuous beam element with six degrees-of-freedom per node for static, buckling and vibration analyses of FG sandwich beams based on a quasi-3D theory. They considered both shear deformation and thickness stretching effect in the analyses.

This study aims to develop an accurate and simpler finite element model based on the first-order shear deformation theory for vibration and buckling of FGBs. Material properties within the beam vary continuously through the thickness according to the power-law form. The beam element proposed here has five nodes and ten degrees-of-freedom. Governing equations of motion are derived by using Lagrange's equations. Accuracy of the element is validated through comparisons with the results available for buckling loads and natural frequencies of FGBs with different end conditions, power-law indices, and span-to-depth ratios.

2. Theory and formulation

2.1. Material properties

Fig. 1 shows an isotropic, nonhomogeneous elastic beam with length L and rectangular cross-section of $b \times h$. The x-, y-, and z-axes are located along the length, width, and height of the beam, respectively. The beam is loaded by an axial compressive force N at its ends. The beam is assumed to be composed of a mixture of two constituents such as ceramic and metal, which are located at its top and bottom surfaces, respectively. Material behavior obeys Hooke's law. The gravity is not considered. Material properties vary continuously through-the-thickness according to the power-law rule given by

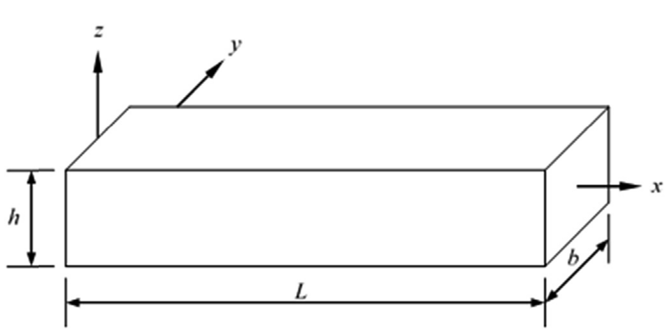

Fig. 1. Geometry and coordinate system of a functionally graded beam.

$$P(z) = (P_m - P_c)\left(\frac{z}{h} + \frac{1}{2}\right)^k + P_m \tag{1}$$

where k is the non-negative power-law exponent, P_m and P_c are the corresponding material properties of the metal and ceramic constituents, e.g., Young's modulus E, Poisson's ratio ν, and mass density ρ, respectively.

2.2. Finite element model

Fig. 2 shows a five-noded beam finite element with four equally spaced nodes and a node at the middle. It has ten degrees-of-freedom including three axial, four transversal and three rotational displacements which are measured at neutral axis of the beam. The nodal displacement vector can thus be given as:

$$\mathbf{u} = \{u_1 \quad u_2 \quad u_3 \quad w_1 \quad w_2 \quad w_3 \quad w_4 \quad \phi_1 \quad \phi_2 \quad \phi_3\}^T \tag{2}$$

where u, w and ϕ are the axial and the transverse displacements, and the total bending rotation of the cross-sections at any point on the neutral axis, respectively. Note that ϕ is assumed to be geometrically unrelated to the slope $\partial w/\partial x$ to account for the shear deformation.

According to the first-order shear deformation theory, the displacement field can be given by

$$\begin{aligned} U(x,z,t) &= u(x,t) - z\,\phi(x,t), \\ W(x,z,t) &= w(x,t) \end{aligned} \tag{3}$$

where t denotes time. The strain-displacement relations are given by

$$\begin{aligned} \varepsilon_{xx} &= \frac{\partial U}{\partial x} = \frac{\partial u}{\partial x} - z\frac{\partial \phi}{\partial x} = u_{,x} - z\phi_{,x}, \\ \gamma_{xz} &= \frac{\partial U}{\partial z} + \frac{\partial W}{\partial x} = \frac{\partial w}{\partial x} - \phi = w_{,x} - \phi \end{aligned} \tag{4}$$

where ε_{xx} and γ_{xz} are the normal and shear strains, respectively. $(\bullet)_{,x}$ denotes the derivative with respect to x. Since the material behavior obeys Hooke's law, the constitutive relations can be written as

$$\sigma_{xx} = E(z)\varepsilon_{xx}, \quad \tau_{xz} = KG(z)\gamma_{xz} \tag{5}$$

where σ_{xx} and τ_{xz} are the normal and shear stresses, respectively. K is the shear correction factor, $E(z)$ is the Young modulus, and $G(z) = E(z)/[2\,(1 + \nu(z))]$ is the shear modulus.

The strain energy of the beam can be given by

$$U = \frac{1}{2}\int_0^L \int_A (\sigma_{xx}\varepsilon_{xx} + \tau_{xz}\gamma_{xz})dA\,dx \tag{6}$$

where A is the cross-sectional area of the beam. Substituting Eqs. (4) and (5) into Eq. (6) yields

$$\begin{aligned} U = \frac{1}{2}\int_0^L \Big[& A_0(u_{,x})^2 - 2A_1 u_{,x}\phi_{,x} + A_2(\phi_{,x})^2 + B_0\Big[(w_{,x})^2 - 2w_{,x}\phi \\ & + (\phi)^2\Big]\Big] dx \end{aligned} \tag{7}$$

where the stiffness coefficients are defined as