ARTICLE IN PRESS
CRAS2B:3490

C. R. Mecanique ••• (••••) •••–•••

Contents lists available at ScienceDirect

Comptes Rendus Mecanique

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Acoustical scattering identification with local impedance through a spectral approach

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ARTICLE INFO

Article history:
Received 24 October 2016
Accepted 18 March 2017
Available online xxxx

Keywords:
Wave finite element
Acoustical propagation
Micro-perforated plate
Locally reacting lining
Scattering

ABSTRACT

Acoustical scattering in waveguides is studied in this paper. The Wave Finite Element (WFE) approach is mainly used, since it allows the reduction of problems dealing with periodic waveguides. The paper deals with guided acoustical propagation, that is, propagation in a main direction is privileged. The scattering by a locally reacting lining is first studied. The liner can be characterised by its local impedance in this case. The equivalent surface impedance is therefore calculated. Then, scattering by a porous layer is considered. A full three-dimensional modelling of the lining is preferred since porous materials are bulk reacting. The scattering matrix of the lined part is computed, and acoustical scattering of high-order modes and conversion between modes are highlighted. The acoustic power attenuation is further evaluated. The response of ducts subjected to constraining boundary conditions is also calculated. Numerical results are presented and compared to those obtained with conventional approaches.

1. Introduction

Reducing noise levels has been one of the main topics dealt with in acoustics. The most standard and readily recognisable technique is to use liners composed essentially of an acoustically absorbing material. However, different liner configurations can be used, depending on the engineering application. The frequency dependence of the acoustic impedance of the liner, and the geometrical characteristics of the liner, which contribute to the expression of the impedance, must be taken into consideration when computing the scattering matrix and the acoustic power attenuation. Different approaches dealt with the acoustic problems using analytical formulations based on the resolution of the Helmholtz equation for canonic examples, or numerical approaches such as Semi-Analytical Finite Element (SAFE) and conventional Finite Element (FE) methods. The Wave Finite Element (WFE) method deals with periodic media. This formulation was firstly developed for photonics [1] and crystal lattice [2] problems. Then, Zhong and Williams [3] extended it for periodic elastic media. The WFE method has been widely used for structural applications: vibrations of uniform waveguide structures [4], Structural Health Monitoring (SHM) and Non-Destructive Testing (NDT) of pipelines [5,6], forced response with harmonic and general load [7], dynamic

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http://dx.doi.org/10.1016/j.crme.2017.03.006
1631-0721/

Please cite this article in press as: M.A. Ben Souf et al., Acoustical scattering identification with local impedance through a spectral approach, C. R. Mecanique (2017), http://dx.doi.org/10.1016/j.crme.2017.03.006

problems of homogeneous thin-walled structures [8], wave propagation in beam-like structures [9], laminates [10], fluid-filled pipes [11], tyres [12] and poroelastic media [13]. The method does not have limitations for the type of element used for modelling. Nevertheless, it may be prone to ill-conditioning issues [14]. Indeed, the cross-sectional mesh density can be raised so that the wave basis is expanded with high-order modes, but time-consuming matrix inversions and ill-conditioned matrices have to be handled while increasing the number of degrees of freedom. For instance, some authors kept only a reduced wave basis while computing the forced response, and the strongly evanescent modes were neglected [15]. The WFE approach was later used for 2D structural applications [16], prediction of the vibro-acoustic behaviour of honeycomb panels [17]. Ben Souf et al. [18] extended the application of the method to media with uncertain parameters and their impact on spectral results.

The scattering matrix can be seen as a characterisation of a lined duct independently of the upstream and downstream conditions [19]. Many works, based on analytical or numerical approaches, focused on the determination of the scattering matrix. Some works, e.g., [20] and [21], used analytical formulations for the computation of the scattering matrix for acoustic problems. In these works, the liner's impedance was assumed to be constant along the duct, and could arbitrarily vary along the circumference. A projection over a rigid duct basis of orthogonal functions was used and terms of coupling between modes due to the scattering were added. Due to the analytical methods constraints, particularly when the geometry is complex, numerical methods were alternatively adopted [22]. These works used a Finite Element method. The scattering matrix of a lined duct was computed for a frequency range such as only uncoupled modes are cut-on, as experiments to extract the acoustic impedance using a pressure source composed of the cut-on modes had to be carried out. In this paper, the WFE method will be used for the rigid parts, and then exploited for computing the acoustical scattering matrix of lined ducts. One elementary cell of the periodic medium is considered. FE commercial packages are used to obtain the mass and the stiffness matrices of the cell. Using the periodicity of the studied medium, a spectral formulation can be written and solved to obtain the wave's characteristics (wavenumbers and deformed shapes). This allows a fairly lower computational cost than when the full FE model is used. The identification of the scattering matrix is based on the hybridisation between the WFE and FE approaches [23], since the lined part of the medium is treated by the conventional FE method. The continuity conditions at the right-hand and the left-hand sides of the coupling element corresponding to the lined part lead to the expression of the scattering matrix.

A well-known solution for reducing noise levels is the use of honeycomb structures. These are particularly used in aircraft engines. The lining is locally reacting and the damping effect can be accurately described by an equivalent impedance at the surface of the duct, that is, simple addition of the acoustic surface impedance of cavities and the acoustic impedance of an associated perforated plate, as detailed in [24–26]. In other engineering applications, porous absorbers can be also used, since they are less frequency dependent. However, it is more convenient to model the entire porous domain rather than using the resulting effective impedance of the wall, as acoustic waves travel also parallel to the walls and an equivalent surface impedance can be only calculated per mode of propagation.

This paper is an extension of the WFE applications to the acoustical guided propagation, and scattering in the presence of acoustic impedance or medium discontinuities. Therefore, different liner configurations are considered. The scattering matrix of the lining is fully calculated considering both propagative and evanescent waves and their conversion. Then, the response of two coupled periodic waveguides with any prescribed boundary conditions (e.g., pressure or velocity excitations, anechoic or partially reflective ends, PML layers...) can be calculated, as well as for single periodic waveguides. An hybrid WFE–FE method can be used while using three-dimensional modelling of the lined part. The considerable advantages of the use of the WFE alternative are reviewed, and the periodicity of the duct parts for which acoustic fields are determined by the WFE method is most likely the only restricting hypothesis of the method.

Notes on the WFE method, empirical models of the acoustic impedance of perforated plates, equivalent fluid theory, and calculation of the acoustic power attenuation are presented in section 2. Numerical examples are dealt with in section 3: Ducts with locally reacting lining are first considered. The effect of the geometric parameters of the liner on the efficiency of the liner is shown. Then, ducts with bulk reacting lining are studied. The considered examples are finally discussed.

2. Theoretical background

2.1. The WFE formulation

The first step of the WFE method is to model a segment of a periodic waveguide, with a same distribution of nodes on its left and right sides, as shown in Fig. 1, say n_d degrees of freedom. The FE packages can be used for modelling. The internal nodes are removed using dynamic condensation. The segment length Δ should be chosen such it is at least six times shorter than the minimal wavelength [14,27]. This segment is governed by the equation:

$$\left(-\omega^2 \boldsymbol{M} + \boldsymbol{K}\right) \boldsymbol{p} = \boldsymbol{D} \boldsymbol{p} = \boldsymbol{v} \tag{1}$$

where $\boldsymbol{p}$ and $\boldsymbol{v}$ are respectively the pressures and particle velocities of the nodes, ω is the angular frequency, and $\boldsymbol{M}$ and $\boldsymbol{K}$ are respectively the mass and stiffness matrices of the segment. Eq. (1) can be rewritten as follows:

$$\begin{pmatrix} \boldsymbol{D}_\mathrm{ll} & \boldsymbol{D}_\mathrm{lr} \\ \boldsymbol{D}_\mathrm{rl} & \boldsymbol{D}_\mathrm{rr} \end{pmatrix} \begin{pmatrix} \boldsymbol{p}_\mathrm{l} \\ \boldsymbol{p}_\mathrm{r} \end{pmatrix} = \begin{pmatrix} \boldsymbol{v}_\mathrm{l} \\ \boldsymbol{v}_\mathrm{r} \end{pmatrix} \tag{2}$$