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Asymptotic curved interface models in piezoelectric composites

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ABSTRACT

We study the electromechanical behavior of a thin interphase, constituted by a piezoelectric anisotropic shell-like thin layer, embedded between two generic three-dimensional piezoelectric bodies by means of the asymptotic analysis in a general curvilinear framework. After defining a small real dimensionless parameter ε, which will tend to zero, we characterize two different limit models and their associated limit problems, the so-called *weak* and *strong* piezoelectric curved interface models, respectively. Moreover, we identify the non-classical electromechanical transmission conditions at the interface between the two three-dimensional bodies.

1. Introduction

Smart materials have been used over the past few decades in several applications in all fields of aeronautical, mechanical, and civil engineering. For what concerns smart structures, the strain state is constantly under control by means of sensors and actuators, usually made of piezoelectric materials, integrated within the structure. The more and more promising applications of piezoelectric composites have lead researchers to develop new methods and analysis tools for a better understanding of the mechanisms and behaviors of such structures, which are subjected to electromechanical interactions. More often, the piezoelectric actuators are obtained by alternating different thin layers of material with highly contrasted electromechanical properties. This generates different types of complex composites, in which each phase interacts with the others.

The asymptotic methods have been successfully applied for the mathematical justification of thin structure models in both fields of elasticity and piezoelectricity, taking into account also thermal and magnetic effects (see, e.g., [1–3]): this has stimulated researchers to tunnel their efforts toward a formal simplification of the modeling of complex structures obtained by joining elements presenting highly contrasted geometrical and mechanical properties. A thin interphase inserted between two generic media can be considered as the most distinctive bonded joint. The asymptotic expansions method allows one to replace the original problem with a reduced transmission problem, in which the thin interphase is substituted by a two-dimensional material surface, i.e. a so-called *imperfect interface*, between the two three-dimensional bodies with non-classical transmission conditions. Within the theory of elasticity, the asymptotic analysis of a thin elastic interphase between two elastic materials has been deeply investigated through the years, by varying the rigidity ratios between the

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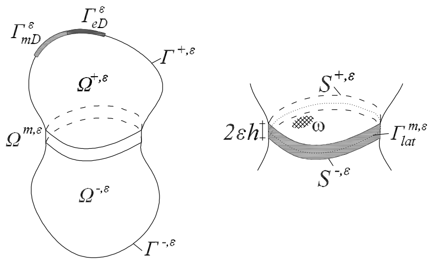

Fig. 1. The geometry of the composite: configuration in the curvilinear coordinates system.

thin inclusion and the surrounding materials and by considering different geometry features (see, e.g., [4–7], within the theory of elasticity, and see [8,9], within the theory of piezoelectricity, including also thermal and magnetic couplings).

This work is conceived as the curvilinear generalization of a previous work [8] on asymptotic planar weak and strong piezoelectric interface models. In the present work, we identify two different interface limit models of a piezoelectric assembly constituted by a thin piezoelectric shell-like layer inserted between two generic piezoelectric bodies by means of an asymptotic analysis in a general curvilinear framework. By defining a small real parameter ε, associated with the thickness and the electromechanical properties of the middle layer, we perform an asymptotic analysis by letting ε tend to zero. We analyze two different situations by varying the electromechanical stiffness ratios between the middle layer and the adherents: namely, the *weak* piezoelectric curved interface, where the electromechanical coefficients of the intermediate domain have order of magnitude ε with respect to those of the surrounding bodies, the *strong* piezoelectric curved interface, where the electromechanical rigidities have order of magnitude $\frac{1}{\varepsilon}$. Within the reduced models, the interphase is replaced by a material surface (*strong* case) or a constraint (*weak* case) whose energy, in both cases, is the limit of the interphase energy. This surface energy is then translated in ad hoc transmission conditions at the interface.

The paper is organized as follows. In Sect. 2, we define the position of the problem and we perform the asymptotic analysis of the problem. In Sect. 3 and Sect. 4, we deduce, respectively, the two limit interface models. Finally, we discuss the results and propose some future developments in the concluding remarks in Sect. 5.

2. Position of the problem and asymptotic expansions

Let Ω^+ and Ω^- be two disjoint open domains with smooth boundaries $\partial\Omega^+$ and $\partial\Omega^-$. Let $\omega := \left\{\partial\Omega^+ \cap \partial\Omega^-\right\}^\circ$ be the interior of the common part of the boundaries which is assumed to be a non-empty domain in $\mathbb{R}^2$ having a positive two-dimensional measure. Let $\boldsymbol{\theta} \in \mathcal{C}^2(\overline{\omega}; \mathbb{R}^3)$ be an immersion such that the two vectors $\mathbf{a}_\alpha(\tilde{x}) := \partial_\alpha \boldsymbol{\theta}(\tilde{x})$ form the covariant basis of the tangent plane to the surface $\boldsymbol{\theta}(\omega)$ at each point $\boldsymbol{\theta}(\tilde{x})$, with $\tilde{x} = (x_\alpha) \in \omega$; the two vectors $\mathbf{a}^\alpha(\tilde{x})$, defined by the relation $\mathbf{a}_\alpha \cdot \mathbf{a}^\beta = \delta^\beta_\alpha$, form the contravariant basis of the tangent plane. Also let $\mathbf{a}_3(\tilde{x}) = \mathbf{a}^3(\tilde{x}) := \frac{\mathbf{a}_1(\tilde{x}) \wedge \mathbf{a}_2(\tilde{x})}{|\mathbf{a}_1(\tilde{x}) \wedge \mathbf{a}_2(\tilde{x})|}$ be the unit normal vector to $\boldsymbol{\theta}(\omega)$. The covariant and contravariant components $a_{\alpha\beta}$ and $a^{\alpha\beta}$ of the first fundamental form, the covariant and mixed components of the second fundamental form, and the Christoffel symbols of the surface are respectively defined by: $a_{\alpha\beta} := \mathbf{a}_\alpha \cdot \mathbf{a}_\beta$, $a^{\alpha\beta} := \mathbf{a}^\alpha \cdot \mathbf{a}^\beta$, $b_{\alpha\beta} := \mathbf{a}^3 \cdot \partial_\beta \mathbf{a}_\alpha$, $b^\beta_\alpha := a^{\beta\sigma} b_{\alpha\sigma}$ and $\Gamma^\sigma_{\alpha\beta} := \mathbf{a}^\sigma \cdot \partial_\beta \mathbf{a}_\alpha$. The covariant derivative of $T^{\alpha\beta}$ are defined by $T^{\alpha\beta}|_\tau := \partial_\tau T^{\alpha\beta} + \Gamma^\alpha_{\beta\sigma} T^{\tau\sigma} + \Gamma^\beta_{\tau\sigma} T^{\alpha\sigma}$.

Let $0 < \varepsilon < 1$ be a dimensionless *small* real parameter. Let us consider $\Omega^{m,\varepsilon} := \omega \times (-\varepsilon h, \varepsilon h)$, $S^{\pm,\varepsilon} := \omega \times \{\pm\varepsilon h\}$ and $\Gamma^{m,\varepsilon}_{lat} := \partial\omega \times (-\varepsilon h, \varepsilon h)$. Let x^ε denote the generic point in the set $\overline{\Omega}^{m,\varepsilon}$ with $x^\varepsilon_\alpha = x_\alpha$. We consider a *shell-like* domain with middle surface $\boldsymbol{\theta}(\overline{\omega})$ and thickness $2\varepsilon h$, whose reference configuration is the image $\boldsymbol{\Theta}^{m,\varepsilon}(\overline{\Omega}^{m,\varepsilon}) \subset \mathbb{R}^3$ of the set $\overline{\Omega}^{m,\varepsilon}$ through the mapping given by $\boldsymbol{\Theta}^{m,\varepsilon}(x^\varepsilon) := \boldsymbol{\theta}(\tilde{x}) + x^\varepsilon_3 \mathbf{a}_3(\tilde{x})$, for all $x^\varepsilon = (\tilde{x}, x^\varepsilon_3) \in \overline{\Omega}^{m,\varepsilon}$.

Moreover, we suppose that there exists an immersion $\boldsymbol{\Theta}^\varepsilon : \overline{\Omega}^\varepsilon \to \mathbb{R}^3$ defined as follows:

$$\boldsymbol{\Theta}^\varepsilon := \begin{cases} \boldsymbol{\Theta}^{\pm,\varepsilon} & \text{on } \overline{\Omega}^{\pm,\varepsilon} \\ \boldsymbol{\Theta}^{m,\varepsilon} & \text{on } \overline{\Omega}^{m,\varepsilon} \end{cases}, \quad \boldsymbol{\Theta}^{\pm,\varepsilon}(S^{\pm,\varepsilon}) = \boldsymbol{\Theta}^{m,\varepsilon}(S^{\pm,\varepsilon}),$$

with $\boldsymbol{\Theta}^{\pm,\varepsilon} : \overline{\Omega}^{\pm,\varepsilon} \to \mathbb{R}^3$ immersions over $\overline{\Omega}^{\pm,\varepsilon}$ defining the curvilinear coordinates on $\overline{\Omega}^{\pm,\varepsilon}$, see Fig. 1. We will note by $g^\varepsilon_{ij} := (\partial^\varepsilon_i \boldsymbol{\Theta}^\varepsilon \cdot \partial^\varepsilon_j \boldsymbol{\Theta}^\varepsilon)$, the covariant components of the metric tensor, with $g^\varepsilon := \det(g^\varepsilon_{ij})$, $\Gamma^{p,\varepsilon}_{ij}$, the Christoffel symbols of the second kind induced by the metric g^ε_{ij} and $T^{ij}\|_k := \partial_k T^{ij} + \Gamma^i_{\ell j} T^{\ell k} + \Gamma^j_{\ell k} T^{\ell i}$, the covariant derivatives of T^{ij}.

Let $(\Gamma^\varepsilon_{mD}, \Gamma^\varepsilon_{mN})$ and $(\Gamma^\varepsilon_{eD}, \Gamma^\varepsilon_{eN})$ be two suitable partitions of $\partial\Omega^\varepsilon := \Gamma^{\pm,\varepsilon} \cup \Gamma^{m,\varepsilon}_{lat}$. The composite is, on the one hand, clamped along Γ^ε_{mD} and at an electrical potential $\varphi^\varepsilon_0 = 0$ on Γ^ε_{eD} and, on the other hand, subject to surface forces $g^{i,\varepsilon}$ on Γ^ε_{mN} and surface electrical charges d^ε on Γ^ε_{eN}. The assembly is also subject to body forces $f^{i,\varepsilon}$ and electrical loadings ρ^ε_e acting in $\Omega^{\pm,\varepsilon}$. The work of the external electromechanical loadings in curvilinear coordinates takes the following form: