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A further analysis on the analogy between friction and plasticity in Solid Mechanics

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ABSTRACT

A strong analogy between plasticity and friction is commonly admitted in the literature while the question of applicability of plasticity principles to frictional contact problems remains open. Besides, the formulations of various friction laws and associated numerical procedures have been derived, mainly based on this analogy. More recently, the well-known asymptotic mechanisms in plasticity, such as shakedown, cyclic plasticity and ratcheting have been shown to possess analogous asymptotic states under cyclic loading on frictional contact interfaces, the relative slip playing the role of plastic strain. The present paper aims at dealing with the problem of bilateral contact with standard friction in order to show the equivalence of this problem with the one of intermediate volume governed by standard plasticity, when the volume tends towards the contact surface. An equivalence theorem is obtained and mathematically proved by an asymptotic analysis leading to localization of plastic strains on a surface. The outcomes of this equivalence theorem for problems governed by standard friction are then presented and the extension to Coulomb's friction is also discussed. A simple example is finally provided to illustrate the main theoretical results of the proved equivalence between both problems.

1. Introduction

The study of classical plasticity has been initiated many years ago and thanks to the pioneering works of Bleich (1932), Drucker (1954), Hill (1950), Koiter (1960), Maier (1969), Melan (1936), Prager (1951), and then to the fundamental contributions of many authors (e.g. Debordes & Nayroles, 1976; Halphen & Nguyen, 1975; Halphen & Salençon, 1987), the plasticity mechanisms are now well-understood and can be realistically modelled. The numerical treatment of classical plasticity has also followed the same successful expanding and industrial finite element codes are at the present apt to provide realistic numerical simulations of problems involving plasticity.

On the other hand, the classical Coulomb's law of friction has been formulated using the fundamental rules of friction, firstly discovered by Amontons (1699), and further developped par Coulomb (1785). Since then the developments regarding friction, and more generally frictional contact problems, have been more mathematically (e.g. Duvaut, 1980; Kikuchi & Oden, 1988; Moreau, 1970) and numerically (e.g. Giannakopoulos, 1989; Wriggers, 2002) oriented considering basically Coulomb's law as the main constitutive model to represent friction.

Although the equivalence is not proved, it is commonly admitted that an obvious analogy exists between both phenomena, considering that the relative tangential slip plays exactly the role of plastic strain. For instance, a frictional patina is

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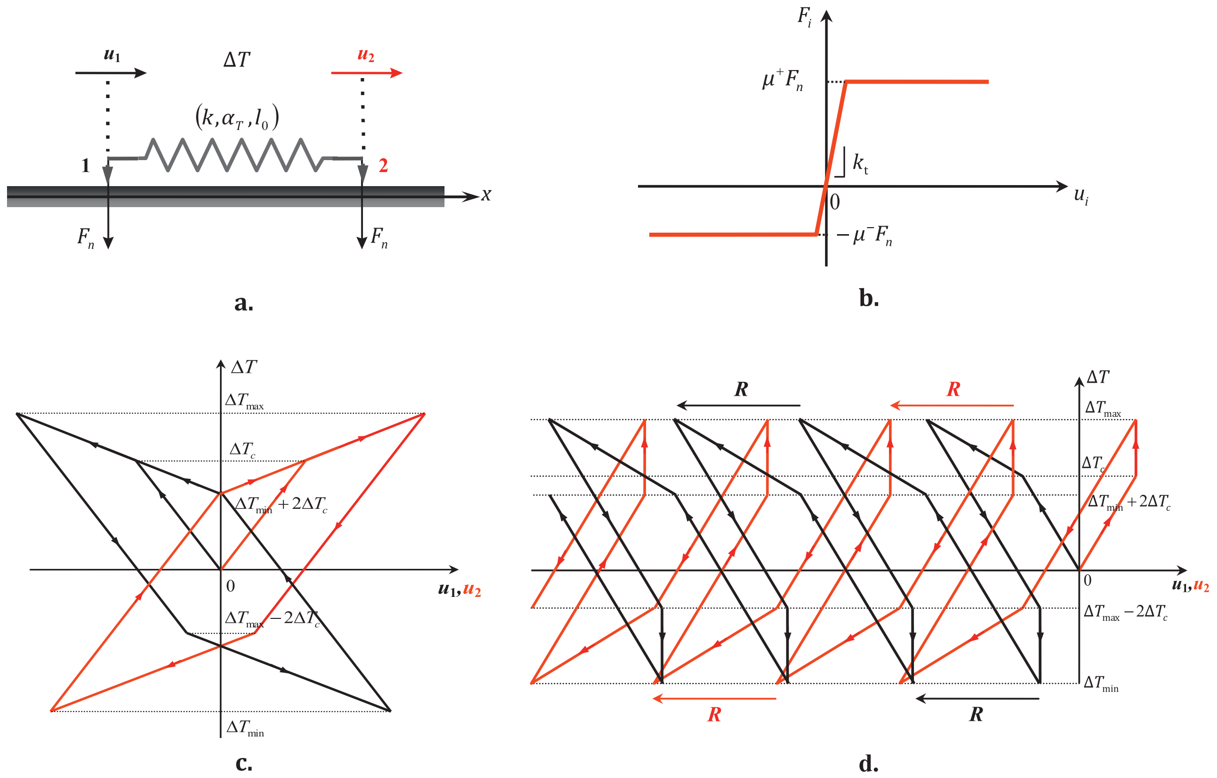

Fig. 1. Example (extract from Antoni et al., 2007) of asymptotic states for one-dimensional elastic discrete model, subjected to quasi-static thermal loading cycles (temperature variation ΔT) (a.) and governed by dissymmetric standard friction (b.) *Slip-shakedown* if $\Delta T_{\max} - \Delta T_{\min} < 2\Delta T_c$ with $\Delta T_c = \min(\mu^-, \mu^+)(F_n/\alpha_T l_0)(1/k + 2/k_t)$, *Cyclic slip* if $\mu^+ = \mu^-$ and $\Delta T_{\max} - \Delta T_{\min} \geq 2\Delta T_c$ (c.), C*umulative slip* in the in the $x < 0$-direction (resp. $x > 0$-direction) if $\mu^- < \mu^+$ (resp. $\mu^- > \mu^+$) and $\Delta T_{\max} - \Delta T_{\min} \geq 2\Delta T_c$ (d.).

commonly used in rheological modeling of perfect plasticity to represent the occurrence of the plastic deformation. The irreversible plastic flow is hence considered comparable to the irreversible relative sliding of contacting solids. The formulation of Coulomb's friction law is also more and more written following classical plasticity formulations. The consideration of this analogy has been firstly studied by Curnier (1984); Drucker (1954); Michalowski and Mróz (1978) and afterwards has arisen in the field of computational mechanics (Wriggers, 1987), followed by several authors (e.g. Giannakopoulos, 1989). Subsequently, it has been highlighted (Antoni, Nguyen, Ligier, Saffre, & Pastor, 2007) that the same kind of asymptotic states can be found to occur on the frictional contact interfaces under cyclic loading, namely slip-shakedown, cyclic slip and cumulative slip (cf. example in Fig. 1), respectively analogous to shakedown, cyclic plasticity and ratcheting in classical plasticity (cf. Koiter, 1960; König, 1987). This analogy in behaviour has pushed many researchers to extend plasticity principles and formulations to frictional contact mechanics.

However, when considering Coulomb-like friction law, this passage is not direct because of the non-associative nature of the law (Drucker, 1954; Michalowski & Mróz, 1978), the normal to Coulomb's frictional cone being not coincident with the irreversible sliding direction (see Fig. 2). As a consequence of this non-associativity, it has been for instance recently demonstrated that: (i) Melan's shakedown theorem does not hold for a pressure-dependent law of friction such as Coulomb's friction. Restrictively; (ii) Melan and Koiter shakedown theorems (Koiter, 1960; Melan, 1936) have been shown to be applicable only under the assumption of no separation and for pressure-independent laws of friction, i.e. if the contact area does not change during the loading phase and if there is no coupling between the contact pressure and the tangential displacement (Antoni & Nguyen, 2008; Barber, Klarbring, & Ciavarella, 2008; Churchman, Korsunsky, & Hills, 2006; Klarbring, Ciavarella, & Barber, 2007). In all those works, the shakedown theorems for frictional contact have been firstly formulated by analogy with plasticity and then their applicability has been either proved defective based on counterexamples for (i) or numerically verified on various examples for (ii). It is worth noting that general theoretical results are still lacking for the shakedown analysis of non-associated laws in plasticity and numerical simulations are generally considered (Bouby, De Saxcé, & Trisch, 2006). Although it is well known that the maximum rate of dissipation criterion can be used as a starting point to derive the flow rules for associative plasticity, it is still possible, as interestingly shown by Srinivasa (2010), to use the maximum rate of dissipation criterion also for pressure-dependent plasticity models (i.e. as for Coulomb's friction). The main difference is that the normality rule is not obtained in stress space but in plastic strain rate space.