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A two-phase self-consistent model for the grid indentation testing of composite materials

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ABSTRACT

The indented composite material is assumed to consist of two phases modeled as hemispherical inhomogeneities embedded at the surface of the semi-infinite homogenized medium with unknown properties. The radii of the hemispherical inhomogeneities are chosen in such a way that the indentation is performed at the center of the corresponding phase site. The effective properties of the homogenized medium are evaluated using the Mori–Tanaka homogenization scheme. Both Poisson's ratios of the phases and the phase concentrations are assumed to be known. The two-phase self-consistent model is based on the first-order asymptotic model for the incremental indentation stiffness of an embedded hemispherical inhomogeneity.

1. Introduction

Determination of effective mechanical properties of multi-heterogeneous materials, including composites (Gibson, 2014; Ulm, Vandamme, Bobko, & Ortega, 2007; Yan, Chung, Wu, & Simon, 2011), microelectronic materials with intermetallic compounds (Albrecht et al., 2003), and reinforced concretes (Němeček, Králík, & Vondřejc, 2013; Silva, Němeček, & Štemberk, 2014), by means of indentation tests along with the intrinsic properties of their constituents is a challenging problem due to the complexity of the real microstructure and the absence of analytical theoretical framework.

Over the last few decades, grid indentation (Albrecht et al., 2005; Němeček, Šmilauer, & Kopecký, 2011; Constantinides, Ravi Chandran, Ulm, & Van Vliet, 2006; Gibson, 2014) has become a widely used technique for direct measurement of the reduced elastic moduli of the heterogeneous constitutes of composite materials, provided the indenter penetration depth, h, is much smaller than the characteristic size of the heterogeneity, D, (Randall, Vandamme, & Ulm, 2009). In what follows, we assume that the characteristic size of a tested heterogenous sample $L = \min\{L_x, L_y, L_z\}$ is sufficiently large to include a representative volume element (RVE). In turn, the characteristic size L_{grid} of the region covered by grid indentation (see Fig. 1) should be larger than the characteristic size of the heterogeneity D, on the one hand, and smaller than L, on the other hand, to reduce the sample-size effect. Note also that the characteristic size of indents, d, is governed by the indentation depth h and depends on the indenter shape (Oliver & Pharr, 2004).

In order to extract information about the material properties from the array of indentation data for a certain grid on the surface of a tested material, a few identification techniques were suggested. The deconvolution technique (Constantinides

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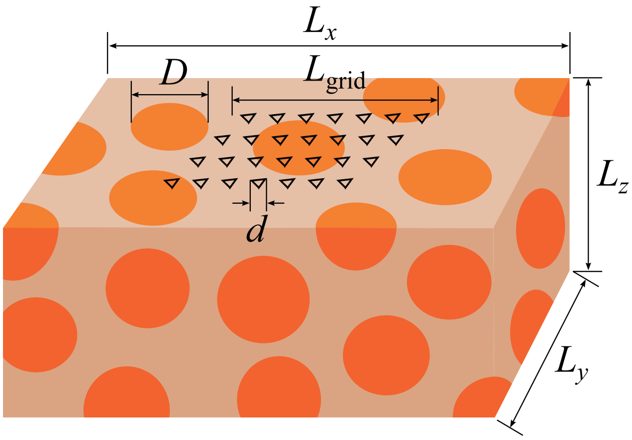

Fig. 1. Schematic of the grid indentation technique for a sample of heterogeneous material.

et al., 2006; Constantinides & Ulm, 2007) assumes that although a few indentations may measure a composite response, the so-called scale separation criterion $h/D < 0.1$ ensures that a large majority of the indentations will probe the intrinsic properties of the individual phases (Randall et al., 2009). In particular, the statistical deconvolution technique (Constantinides et al., 2006; Ulm et al., 2007) provides mean property values, volume fractions, and standard deviations of the phase mechanical properties (elastic modulus and hardness) by fitting the theoretical probability density functions (PDF) to the experimental normalized histogram of the measured quantity. The self-consistent indentation technique (Randall et al., 2009) yields the homogenized indentation modulus, $M_{\rm hom}$, by introducing a "virtual" composite material, which is composed of N phases characterized by N indentation moduli $\{M_j\}_{j=1,\dots,N}$. The finite element simulation based techniques were proposed in a number of works (Albrecht et al., 2005; Jang, Kim, Gibson, & Suhr, 2013; Leisen, Kerkamm, Bohn, & Kamiah, 2012; Yan et al., 2011).

It should be emphasized that none of the methods for evaluating grid indentation data mentioned above takes into account the effect of mechanical interaction of the indented phase with the surrounding phases. This effect was assessed by Constantinides et al. (2006) using the first-order perturbation model (Gao, Chiu, & Lee, 1992) and the semi-analytical model (Perriot & Barthel, 2004) developed for the case of an elastic layer bonded to an elastic half-space.

In the present paper, we develop a two-phase self-consistent model, based on the recently introduced small-scale indentation asymptotic model (Argatov & Sabina, 2015) as well as on the self-consistent approach, which is a frequently used method to evaluate effective mechanical properties of heterogeneous media (Budiansky, 1965; Dvorak, 1982; Hill, 1965; Kanaun & Levin, 2008; Sabina et al., 2015; Sabina & Willis, 1988). The main goal of the present paper is to address the interaction effect of the material phases and, thereby, to improve the efficiency of grid indentation testing.

2. Grid indentation of composites (necessary data collected)

2.1. Indentation elastic modulus

The grid indentation (Constantinides et al., 2006) is a method of performing a large number of indentations located on a grid defined by a grid spacing that is larger than the characteristic size of the indent (indentation impression). As a result of each indentation, one can measure the increment indentation stiffness

$$S = \frac{dP}{dh}\bigg|_{h=h_{\max}}. \tag{1}$$

Here, P is the contact force, h is the indenter displacement, and $h_{\max}$ is the maximum indentation depth. In order to separate the effect of plastic deformations, the initial part of the unloading force-displacement curve is used (Bulychev, Alekhin, Shorshorov, & Ternovskii, 1976; Oliver & Pharr, 1992).

In the case of frictionless indenting an isotropic homogeneous elastic half-space (with the shear modulus G and Poisson's ratio ν), the indentation stiffness is related to the elastic properties and the contact geometry according to the generalized BASh (Bulychev–Alekhin–Shorshorov) formula as follows:

$$\frac{dP}{dh} = 2\beta\sqrt{\frac{A}{\pi}}E_r. \tag{2}$$

Here, A is the contact area, β is the contact area shape factor, $E_r = 2G/(1-\nu)$ is the so-called reduced (or effective) elastic modulus (which is evaluated by neglecting the indenter deformation). Note that formula (2) was originally introduced