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A new analysis of stresses in arteries based on an Eulerian formulation of growth in tissues

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ABSTRACT

The simple analysis of stresses in arteries considers the artery as an incompressible elastic circular cylindrical tube. When the intact artery is unloaded and cut transmurally, it springs open to a nearly circular cylindrical sector. The resulting nonzero opening angle indicates the existence of residual stresses in the unloaded intact artery. This paper presents a new analysis of stresses in arteries based on an Eulerian formulation of elastic deformations in soft tissues that models the inelastic process of remodeling (homeostasis) towards its homeostatic state. Within the context of this new analysis it is not necessary to model details of homeostasis. Instead, it is possible to directly determine the loaded homeostatic state, which is considered as the reference configuration of the artery, and to specify the geometry and associated values of the circumferential and axial stresses by matching measurements of the geometry of the unloaded open artery. The predicted geometry of the unloaded intact artery is shown to be accurate relative to the measurements. Also, predictions of the stress distributions for remodeling due to hypertension are presented.

1. Introduction

The analysis of stresses in arteries has been an active field of research for decades. The artery is essentially a tube structure with three concentric layers called the intima, media and adventitia (e.g., Holzapfel, Gasser, & Ogden, 2000). The relative concentrations of collagen fibers, elastin fibers, smooth muscle cells, etc., vary in each of these layers.

The seminal works on stresses in arteries (Chuong & Fung, 1986; Fung, 1990) model the intact artery as a thick circular cylindrical tube with uniform elastic properties through its thickness. The intact artery of inner and outer radii, a and b, respectively, is loaded by internal pressure. Chuong and Fung (1986) measured the geometry $\{a = a_0,\ b = b_0\}$ of the unloaded intact thoracic artery of a cat. They also cut the artery transmurally and measured the geometry $\{a = a_O,\ b = b_O\}$ and the opening angle ω of the unloaded open artery (see Fig. 1). A nonzero value of the opening angle indicates the existence of residual stresses in the unloaded intact artery, with its inner and outer regions being in compression and tension, respectively. To estimate the stress distribution in the intact loaded artery, Chuong and Fung (1986) specified the unloaded open artery as a stress-free reference configuration and assumed incompressible response with a hyperelastic orthotropic strain energy function. Specifically, the material constants for a rabbit thoracic artery were used together with the unloaded geometry of the thoracic artery of a cat. This strain energy function was reexamined in Humphrey (1999) and shown to be well suited for modeling stresses in arteries.

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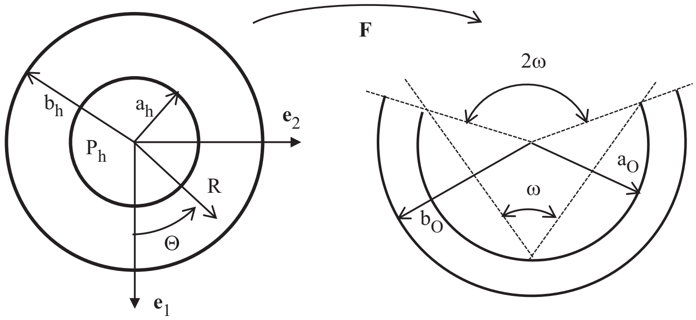

Fig. 1. Sketch of an artery in its homeostatic state at pressure P_h and the unloading process to obtain the opening angle ω.

It is now well known that the unloaded open artery is not stress-free pointwise (e.g. Holzapfel, Sommer, Auer, Regitnig, & Ogden, 2007; Rachev, 1997; Rachev & Greenwald, 2003; Zhou & Lu, 2009) and that the remodeling process is not uniform through the artery's thickness (e.g. Rachev & Gleason, 2011; Schroder & Brinkhues, 2014). Hyperelastic models that include residual stresses in the chosen reference configuration are discussed in e.g., Ahamed, Dorfmann, and Ogden (2016), Merodio and Ogden (2016) and Merodio, Ogden, and Rodriguez (2013).

From the point of view of continuum mechanics, it is natural to model the artery using mixture theory (e.g. Ambrosi et al., 2011; Ateshian and Humphrey, 2012; Sciume et al., 2013). However, for general mixtures it is necessary to determine the flow field of each constituent and to specify constitutive equations for their interactions. These complications make the use of general mixture theory difficult and computationally intensive. Humphrey and Rajagopal (2002) proposed a constrained mixture model with no relative motion between the constituents.

Taber and Humphrey (2001) suggest that material properties vary transmurally through the artery. Alford, Humphrey, and Taber (2008) present a comprehensive study of a thick walled tube using a constrained mixture theory with components characterizing collagen, elastin and muscle. The model includes evolution equations for stress-dependent growth, muscle activation as well as heterogeneity due to different material properties in the media and adventitia. This model is based on the Lagrangian formulation of growth proposed by Rodriguez, Hoger, and McCulloch (1994), which expresses the total deformation gradient **F** from a reference configuration in the multiplicative form

$$\mathbf{F} = \mathbf{F}_e \mathbf{F}_g. \tag{1.1}$$

In this expression, the growth deformation $\mathbf{F}_g$ characterizes inelastic deformation from the reference configuration to a stress-free intermediate configuration and $\mathbf{F}_e$ is the elastic deformation from the intermediate configuration to the present configuration.

Taber (1995) presented a review of the literature related to growth, remodeling and morphogenesis of biological tissues. More recent reviews of growth in living systems can be found in Ambrosi et al. (2011) and Kuhl (2014). Remodeling is often modeled by proposing an evolution equation for $\mathbf{F}_g$ (e.g. Rachev, Stergiopulos, & Meister, 1996, 1998; Saez, Pena, Martinez, & Kuhl, 2014; Taber, 1998).

Recently, Rubin, Safadi, and Jabareen (2015) developed Eulerian evolution equations for an elastic dilatation J_e and a unimodular tensorial measure $\mathbf{B}'_e$ of elastic distortional deformation for growth of soft tissues. Specifically, this new theory models growth with homeostasis being an inelastic process that causes the elastic deformations measures $\{J_e, \mathbf{B}'_e\}$ to approach their homeostatic values $\{J_h, \mathbf{H}'\}$. Humphrey and Rajagopal (2002) proposed an evolving natural stress-free configuration to characterize growth and remodeling of soft tissues. Since the mapping from an arbitrary reference configuration to the natural stress-free configuration is not necessarily compatible, the natural configuration usually cannot be realized physically. Here and in Rubin et al. (2015), the term homeostatic state is used to characterize the local state of the material when $\{J_e, \mathbf{B}'_e\}$ equal their homeostatic values $\{J_h, \mathbf{H}'\}$. Reference to a local state of the material attempts to emphasize that the homeostatic state of the material is not necessarily connected to any stress-free configuration since the collection of homeostatic states can be inhomogeneous and not determined by any compatible deformation field from a uniformly stress-free reference configuration.

The constrained theory in Humphrey and Rajagopal (2002) includes interactions of the components in the tissue which can be used to motivate constitutive equations for the mixture being modeled as a simple continuum with a single velocity field. This simplified model could be developed within the context of the Eulerian formulation in Rubin et al. (2015), which also considers a single velocity field. In this formulation, the growing tissue is considered as an open system with external rates of mass and energy supply due to mechanobiological processes. Consequently, it presents a simplified model of flow processes that cause growth.

Hypertension causes a process of remodeling (homeostasis) with the artery approaching a homeostatic state. Since homeostasis occurs over long time periods relative to the normal cardiac cycle, the homeostatic state is static even though cells are continually changing. Hypertension also causes an increase in the diastolic and systolic pressures P_d and P_s, respectively. Fig. 1 shows a sketch of an artery in its homeostatic state with $\{a = a_h,\ b = b_h,\ P = P_h\}$.