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ORIGINAL ARTICLE

Estimation for stochastic volatility model: Quasi-likelihood and asymptotic quasi-likelihood approaches

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Abstract For estimation of the stochastic volatility model (SVM), this paper suggests the quasi-likelihood (QL) and asymptotic quasi-likelihood (AQL) methods. The QL approach is quite simple and does not require full knowledge of the likelihood functions of the SVM. The AQL technique is based on the QL method and is used when the covariance matrix $\mathbf{\Sigma}$ is unknown. The AQL approach replaces the true variance–covariance matrix $\mathbf{\Sigma}$ by nonparametric kernel estimator of $\mathbf{\Sigma}$ in QL.
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1. Introduction

Consider the stochastic volatility process y_t which satisfies the stochastic volatility model

$$y_t = e^{\frac{h_t}{2}}\eta_t, \quad t = 1, 2, 3, \ldots, T \tag{1.1}$$

and

$$h_t = \gamma + \phi h_{t-1} + \delta_t, \quad t = 1, 2, 3, \ldots, T. \tag{1.2}$$

Furthermore, η_t are independent and identically distributed (i.i.d) with $E(\eta_t) = 0$ and $V(\eta_t) = \sigma^2_\eta$, and δ_t are i.i.d with $E(\delta_t) = 0$ and $V(\delta_t) = \sigma^2_\delta$. For estimation and application of the stochastic volatility model (SVM) (see Jacquire et al., 1994; Breidt and Carriquiry, 1996; Sandmann and Koopman, 1998; Pitt and Shepard, 1999; Papanastastiou and Ioannides, 2004; Alzghool and Lin, 2008; Chan and Grant, 2015; Pinho et al., 2016) Sandmann and Koopman (1998) introduced the Monte Carlo maximum-likelihood procedure. Davis and Rodriguez-Yam (2005) proposed another estimation technique that relies on the likelihood function.

This paper applies the quasi-likelihood (QL) and asymptotic quasi-likelihood (AQL) approaches to SVM. The QL approach relaxes the distributional assumptions but has a restriction that assumes that the conditional variance process is known. To overcome this limitation, we suggest a substitute technique, the AQL methodology, merging the kernel technique used for parameter estimation of the SVM. This AQL

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Table 1 The QL and AQL estimates; the RMSE of each estimate is given below that estimate.

	γ	ϕ	μ	σ_δ	σ_ϵ	γ	ϕ	μ	σ_δ	σ_ϵ
True	−0.821	0.90	−1.271	0.675	2.22	-0.411	0.95	−1.271	0.484	2.22
QL	−0.809	0.901	−1.366	0.344	2.15	-0.417	0.950	−1.144	0.382	2.05
	0.108	0.013	0.157	0.331	0.123	0.080	0.010	0.147	0.104	0.205
AQL	−0.821	0.896	−1.257	0.330	2.34	-0.429	0.943	−1.360	0.342	2.25
	0.108	0.015	0.088	0.158	0.347	0.085	0.014	0.120	0.111	0.148
True	−0.736	0.90	−1.271	0.363	2.22	-0.368	0.95	−1.271	0.260	2.22
QL	−0.889	0.881	−1.199	0.321	2.02	-0.511	0.931	−1.185	0.318	2.01
	0.176	0.022	0.099	0.046	0.23	0.159	0.021	0.098	0.061	0.23
AQL	−0.850	0.876	−1.279	0.293	2.16	−0.496	0.927	−1.284	0.309	2.16
	0.231	0.038	0.051	0.089	0.124	0.181	0.030	0.049	0.063	0.129
True	−0.706	0.90	−1.271	0.135	2.22	-0.353	0.95	−1.271	0.096	2.22
QL	−0.695	0.905	−1.043	0.040	2.21	-0.364	0.946	−1.660	0.070	2.17
	0.017	0.006	0.247	0.095	0.12	0.019	0.006	0.404	0.026	0.13
AQL	−0.889	0.872	−1.111	0.28	2.09	−0.504	0.927	−1.125	0.295	2.10
	0.329	0.049	0.164	0.153	0.164	0.224	0.034	0.150	0.167	0.202
True	−0.147	0.98	−1.271	0.166	2.22	-0.141	0.98	−1.271	0.061	2.22
QL	−0.169	0.977	−1.327	0.072	2.23	-0.140	0.979	−1.705	0.018	2.22
	0.027	0.004	0.155	0.094	0.12	0.003	0.001	0.450	0.043	0.12
AQL	−0.225	0.965	−1.342	0.316	2.13	−0.238	0.961	−1.336	0.310	2.11
	0.109	0.019	0.083	0.130	0.15	0.125	0.023	0.074	0.156	0.251

methodology enables a substitute technique for parameter estimation when the conditional variance process is unknown.

This paper is structured as follows. The QL and AQL approaches are introduced in Section 2. The SVM estimation using the QL and AQL methods, reports of simulation outcomes, and numerical cases are presented in Section 3. The QL and AQL techniques are applied to a real data set in Section 4. Section 5 summarizes and concludes the paper.

2. The QL and AQL methods

In this section, we introduce the QL and AQL methods.

2.1. The QL Method

Let the observation equation be given by

$$\mathbf{y}_t = \mathbf{f}_t(\boldsymbol{\theta}) + \zeta_t, \quad t = 1, 2, 3 \ldots, T. \tag{2.1.1}$$

ζ_t is a sequence of martingale difference with respect to $\mathcal{F}_t, \mathcal{F}_t$ denotes the σ-field generated by $\mathbf{y}_t, \mathbf{y}_{t-1}, \ldots, \mathbf{y}_1$ for $t \geqslant 1$; that is, $E(\zeta_t|\mathcal{F}_{t-1}) = E_{t-1}(\zeta_t) = 0$; where $\mathbf{f}_t(\boldsymbol{\theta})$ is an $\mathcal{F}_{t-1}$ measurable; and $\boldsymbol{\theta}$ is a parameter vector, which belongs to an open subset $\Theta \in R^d$. Note that $\boldsymbol{\theta}$ is a parameter of interest. We assume that $E_{t-1}(\zeta_t \zeta_t') = \boldsymbol{\Sigma}_t$ is known. Now, the liner class $\mathcal{G}_T$ of the estimating function (EF) can be defined by

$$\mathcal{G}_T = \left\{ \sum_{t=1}^{T} \mathbf{W}_t(\mathbf{y}_t - \mathbf{f}_t(\boldsymbol{\theta})) \right\}$$

and the quasi-likelihood estimation function (QLEF) can be defined by

$$\mathbf{G}_T^*(\boldsymbol{\theta}) = \sum_{t=1}^{T} \dot{\mathbf{f}}_t(\boldsymbol{\theta}) \boldsymbol{\Sigma}_t^{-1} (\mathbf{y}_t - \mathbf{f}_t(\boldsymbol{\theta})), \tag{2.1.2}$$

where $\mathbf{W}_t$ is $\mathcal{F}_{t-1}$-measurable and $\dot{\mathbf{f}}_t(\boldsymbol{\theta}) = \partial \mathbf{f}_t(\boldsymbol{\theta})/\partial \boldsymbol{\theta}$. Then, the estimation of $\boldsymbol{\theta}$ by the QL method is the solation of the QL equation $\mathbf{G}_T^*(\boldsymbol{\theta}) = 0$ (see Hedye, 1997).

If the sub-estimating function spaces of $\mathcal{G}_T$ are considered as follows,

$$\mathcal{G}_t = \{\mathbf{W}_t(\mathbf{y}_t - \mathbf{f}_t(\boldsymbol{\theta})), \ t = 1, 2, 3 \ldots, T\},$$

then the QLEF in the space $\mathcal{G}_t$ can be defined by

$$\mathbf{G}_{(t)}^*(\boldsymbol{\theta}) = \dot{\mathbf{f}}_t(\boldsymbol{\theta}) \boldsymbol{\Sigma}_t^{-1} (\mathbf{y}_t - \mathbf{f}_t(\boldsymbol{\theta})) \tag{2.1.3}$$

and the estimation of $\boldsymbol{\theta}$ by the QL method is the solation of the QL equation $\mathbf{G}_{(t)}^*(\boldsymbol{\theta}) = 0$.

A limitation of the QL method is that the nature of $\boldsymbol{\Sigma}_t$ may not be obtainable. A misidentified $\boldsymbol{\Sigma}_t$ could result in a deceptive inference about parameter $\boldsymbol{\theta}$. In the next subsection, we introduce the AQL method, which is basically the QL estimation assuming that the covariance matrix $\boldsymbol{\Sigma}_t$ is unknown.

2.2. The AQL method

The QLEF (see (2.1.2) and (2.1.3)) relies on the information of Σ_t. Such information is not always accessible. To find the QL when $E_{t-1}(\zeta_t \zeta_t')$ is not accessible, Lin (2000) proposed the AQL method.

Definition 2.2.1. Let $\mathbf{G}_{T,n}^*$ be a sequence of the EF in $\mathcal{G}$. For all $\mathbf{G}_T \in \mathcal{G}$, if

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