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Semiclassical limits of ground states for Hamiltonian elliptic system with gradient term[☆]

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ABSTRACT

In this paper, we study the following Hamiltonian elliptic system with gradient term

$$\begin{cases} -\epsilon^2 \Delta\psi + \epsilon\vec{b}\cdot\nabla\psi + \psi + V(x)\varphi = \displaystyle\sum_{i=1}^{I} K_i(x)|\eta|^{p_i-2}\varphi \ \ in\ \mathbb{R}^N, \\ -\epsilon^2 \Delta\varphi - \epsilon\vec{b}\cdot\nabla\varphi + \varphi + V(x)\psi = \displaystyle\sum_{i=1}^{I} K_i(x)|\eta|^{p_i-2}\psi \ \ in\ \mathbb{R}^N, \end{cases}$$

where $\eta = (\psi,\varphi) : \mathbb{R}^N \to \mathbb{R}^2$, $V, K_i \in C(\mathbb{R}^N,\mathbb{R})$, $\epsilon > 0$ is a small parameter and $\vec{b}$ is a constant vector. Suppose that V is sign-changing and has at least one global minimum, and K_i has at least one global maximum. We prove that there are two families of semiclassical solutions, for sufficiently small ϵ, with the least energy, one concentrating on the set of minimal points of V and the other on the set of maximal points of K_i. Moreover, the convergence and exponential decay of semiclassical solutions are also explored.

1. Introduction and main results

We study the following Hamiltonian elliptic system with gradient term

$$\begin{cases} -\epsilon^2 \Delta\psi + \epsilon\vec{b}\cdot\nabla\psi + \psi + V(x)\varphi = \displaystyle\sum_{i=1}^{I} K_i(x)|\eta|^{p_i-2}\varphi \ \ \text{in } \mathbb{R}^N, \\ -\epsilon^2 \Delta\varphi - \epsilon\vec{b}\cdot\nabla\varphi + \varphi + V(x)\psi = \displaystyle\sum_{i=1}^{I} K_i(x)|\eta|^{p_i-2}\psi \ \ \text{in } \mathbb{R}^N, \end{cases} \qquad (\mathcal{P}_\epsilon)$$

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where $\eta = (\psi, \varphi) : \mathbb{R}^N \to \mathbb{R}^2$, $p_i \in (2, 2^*)$, ϵ is a small positive parameter, $\vec{b} = (b_1, \ldots, b_N)$ is a constant vector, V can be sign-changing and $\inf K_i > 0$. Here, 2^* denotes the usual critical exponent for $N \geq 3$. In this paper, we are concerned with the existence, exponential decay and concentration phenomenon of semiclassical ground state solutions of problem $(\mathcal{P}_\epsilon)$.

Systems $(\mathcal{P}_\epsilon)$ or similar to $(\mathcal{P}_\epsilon)$ were studied by a number of authors. But most of them focused on the case $\vec{b} = \vec{0}$. For example, see [1–14] and the references therein. When $\vec{b} \neq \vec{0}$ and $\epsilon = 1$, there are not many works on elliptic systems with gradient term. Zhao and Ding [15] considered the following system

$$\begin{cases} -\Delta\psi + \vec{b}(x) \cdot \nabla\psi + V(x)\psi = H_\varphi(x, \psi, \varphi) & \text{in } \mathbb{R}^N, \\ -\Delta\varphi - \vec{b}(x) \cdot \nabla\varphi + V(x)\varphi = H_\psi(x, \psi, \varphi) & \text{in } \mathbb{R}^N, \end{cases} \tag{1.1}$$

where $\vec{b}(x) = (b_1(x), \ldots, b_N(x)) \in C^1(\mathbb{R}^N, \mathbb{R}^N)$, $V \in C(\mathbb{R}^N, \mathbb{R})$ and $H \in C^1(\mathbb{R}^N \times \mathbb{R}^2, \mathbb{R})$. In this case, the appearance of the gradient term in this system will bring some difficulties, and the variational framework for the case $\vec{b} = \vec{0}$ cannot work any longer. Hence the authors first established suitable variational framework through the studying of the spectrum of operator, and obtained the multiplicity of solution for the non-periodic asymptotically quadratic case by applying the generalized linking theorem. Moreover, without the assumption that $H(x, \eta)$ is even in η, infinitely many geometrically distinct solutions for the periodic asymptotically quadratic case were obtained by using a reduction method. For the periodic superquadratic case, Zhang et al. [16] proved the existence of ground state solution for system (1.1). Recently, Yang et al. [17] considered the non-periodic superquadratic system

$$\begin{cases} -\Delta\psi + \vec{b} \cdot \nabla\psi + \psi = H_\varphi(x, \psi, \varphi) & \text{in } \mathbb{R}^N, \\ -\Delta\varphi - \vec{b} \cdot \nabla\varphi + \varphi = H_\psi(x, \psi, \varphi) & \text{in } \mathbb{R}^N, \end{cases} \tag{1.2}$$

with a constant vector $\vec{b}$. Since the problem is set in unbounded domain with non-periodic nonlinearities, the $(C)_c$-condition does not hold in general. To overcome the difficulties, they first considered certain limit problem related to system (1.2) which is autonomous, and constructed linking levels of the variational functional and proved $(C)_c$-condition.

For small $\epsilon > 0$ the solutions (standing waves) of $(\mathcal{P}_\epsilon)$ are referred to as semiclassical states, which describes the transition from quantum mechanics to classical mechanics when the parameter ϵ goes to zero, and possess an important physical interest. In this framework, from a mathematical viewpoint, one is interested not only in existence of solutions but also in their asymptotic behavior as $\epsilon \to 0$. Typically, solutions tend to concentrate around critical points of the linear potentials or the nonlinear potentials: such solutions are called spikes. However, to the best of our knowledge, there is only a little works concerning the existence and concentration phenomenon of semiclassical states. Very recently, Zhang et al. [18] considered the following singularly perturbed system

$$\begin{cases} -\epsilon^2\Delta\psi + \epsilon\vec{b} \cdot \nabla\psi + \psi = K(x)|\eta|^{p-2}\varphi & \text{in } \mathbb{R}^N, \\ -\epsilon^2\Delta\varphi - \epsilon\vec{b} \cdot \nabla\varphi + \varphi = K(x)|\eta|^{p-2}\psi & \text{in } \mathbb{R}^N, \end{cases} \tag{1.3}$$

with $p \in (2, 2^*)$. The authors proved that (1.3) has a semiclassical ground state solutions, which concentrates around the maxima point of nonlinear potential K. Since the phenomenon of concentration is very interesting both in mathematics and physics. Thus, in the present paper, we shall continue to study the existence, exponential decay and concentration phenomena of semiclassical ground state solutions for system $(\mathcal{P}_\epsilon)$. Here, it is worth mentioning that, as to the concentration of semiclassical ground state solutions of system $(\mathcal{P}_\epsilon)$, there is competition between the linear potential V and the nonlinear potential K_i, i.e., V want to attract ground state solutions to its minimum points but K_i want to attract ground state solutions to its maximum points. Also the solutions depend not only on the linear potential but also on the nonlinear potential. Hence,