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Large time behavior of solutions to a fully parabolic attraction–repulsion chemotaxis system with logistic source

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ABSTRACT

This paper deals with an attraction–repulsion chemotaxis system with logistic source

$$\begin{cases} u_t = \Delta u - \chi\nabla\cdot(u\nabla v) + \xi\nabla\cdot(u\nabla w) + f(u), & x\in\Omega,\ t>0,\\ v_t = \Delta v + \alpha u - \beta v, & x\in\Omega,\ t>0,\\ w_t = \Delta w + \gamma u - \delta w, & x\in\Omega,\ t>0 \end{cases}$$

under homogeneous Neumann boundary conditions in a smooth bounded domain $\Omega\subset\mathbb{R}^N$ $(N\geq 1)$, where parameters χ, ξ, α, β, γ and δ are positive and $f(s)=\kappa s-\mu s^{1+k}$ *with* $\kappa\in\mathbb{R}$, $\mu>0$ *and* $k\geq 1$. It is shown that the corresponding system possesses a unique global bounded classical solution in the cases $k>1$ or $k=1$ with $\mu>C_N\mu^*$ for some $\mu^*, C_N>0$. Moreover, the large time behavior of solutions to the problem is also investigated. Specially speaking, when $\kappa<0$ (resp. $\kappa=0$), the corresponding solution of the system decays to $(0,0,0)$ exponentially (resp. algebraically), and when $\kappa>0$ the solution converges to $\left(\left(\frac{\kappa}{\mu}\right)^{1/k}, \frac{\alpha}{\beta}\left(\frac{\kappa}{\mu}\right)^{1/k}, \frac{\gamma}{\delta}\left(\frac{\kappa}{\mu}\right)^{1/k}\right)$ exponentially if μ is larger.

1. Introduction

This paper is concerned with the following attraction–repulsion chemotaxis system

$$\begin{cases} u_t = \Delta u - \chi\nabla\cdot(u\nabla v) + \xi\nabla\cdot(u\nabla w) + f(u), & x\in\Omega,\ t>0,\\ \tau v_t = \Delta v + \alpha u - \beta v, & x\in\Omega,\ t>0,\\ \tau w_t = \Delta w + \gamma u - \delta w, & x\in\Omega,\ t>0,\\ \dfrac{\partial u}{\partial\nu} = \dfrac{\partial v}{\partial\nu} = \dfrac{\partial w}{\partial\nu} = 0, & x\in\partial\Omega,\ t>0,\\ u(x,0)=u_0(x), \qquad \tau v(x,0)=v_0(x), \qquad \tau w(x,0)=w_0(x), & x\in\Omega \end{cases} \tag{1.1}$$

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in a bounded domain $\Omega \subset \mathbb{R}^N$ ($N \geq 1$) with smooth boundary $\partial\Omega$. The model (1.1) was proposed in [1] to describe the aggregation of microglia in Alzheimer's disease, where unknown functions u, v and w denote the concentrations of Microglia, chemoattractant and chemorepellent which are produced by Microglia, respectively. The positive parameters χ and ξ are the chemotactic coefficients, α, β, γ and δ are chemical production and degradation rates. Here $\tau = 0, 1$ is parameter, and the logistic source f describes the cell proliferation and death. It is noticed that (1.1) was also introduced in [2] to describe the quorum sensing effect in the chemotactic process.

The first two equations of (1.1) with $\xi = 0$ comprise the Keller–Segel chemotaxis system

$$\begin{cases} u_t = \Delta u - \chi\nabla\cdot(u\nabla v) + f(u), & x \in \Omega,\ t > 0, \\ \tau v_t = \Delta v + \alpha u - \beta v, & x \in \Omega,\ t > 0. \end{cases} \tag{1.2}$$

The most characteristic ingredient of this system is that it reflect the attractive chemotaxis through a nonlinear cross-diffusive term. Since the blow-up is an extreme facet of bacterial aggregation, considerable efforts have been devoted to identifying circumstances under which explosions are precluded and thus generates pattern formation which is applicable in realistic situation (see [3–12]). It is verified that the logistic source $f(u) = \mu u(1-u)$ with $\mu > 0$ exerts a certain dampening influence which may suppress blow-up in many relevant situations: all of the solutions to (1.2) remain uniformly bounded globally in two dimensional bounded domain [13,14], alternatively in higher dimensions ($N \geq 3$), the uniform-in-time boundedness of classical solutions to (1.2) with $\tau = 0, 1$ is valid when $f(u) \leq a - \mu u^2$ for some $a \geq 0$ and suitably large $\mu > 0$ [14,15]. In particular, a more generalized logistic term $f(u) = \kappa u - \mu u^2$ with $\kappa \in \mathbb{R}$ and $\mu > 0$ was considered in [3], where the existence of global weak solution to a three-dimensional chemotaxis system was proved for arbitrarily small values of $\mu > 0$, which becomes the classical solution after some time if κ is not too large. As source term f is controlled by $-c_0(u + u^\alpha)$ and $a - \mu u^\alpha$ with some $a \geq 0, \mu, c_0 > 0$ and $\alpha < 2$, the uniform-in-time boundedness of some weak solutions to (1.2) with $\tau = 1$ is also shown in the recent paper [16]. It should be mentioned that the blow-up is possible in a slightly modified version of system (1.2) with logistic source [17].

On the other hand, in the absence of chemoattractant, the first and third equations of (1.1) with $\chi = 0$ form a repulsive Keller–Segel model

$$\begin{cases} u_t = \Delta u + \xi\nabla\cdot(u\nabla w) + f(u), & x \in \Omega,\ t > 0, \\ \tau w_t = \Delta w + \gamma u - \delta w, & x \in \Omega,\ t > 0. \end{cases} \tag{1.3}$$

It should be mentioned that based on a Lyapunov functional identity, Cieślak, Laurençot and C. Morales-Rodrigo [18] showed the global existence of classical solutions to (1.3) in two dimensions and weak solutions in three and four dimensions.

Compared to the classical chemotaxis model, the mathematical analysis on the boundedness of the attraction–repulsion chemotaxis model (1.1) is much harder due to the complicated interactions between three species u, v and w and the difficulty of constructing a Lyapunov functional. For the model (1.1) with $\tau = 0$ and $f(u) \equiv 0$, Tao and Wang [19] obtained the global boundedness of the solution in high-dimensions if $\xi\gamma > \chi\alpha$; while if $\xi\gamma < \chi\alpha$, the solution might blow up in the two-dimensional case if the cell mass is larger than a threshold number. Recently, Jin and Wang [20] investigated the boundedness, blowup and critical mass phenomenon in the parabolic–parabolic–elliptic case of model (1.1) (i.e., $\tau = 1$ in the second equation of (1.1), while $\tau = 0$ in the third one). As for the model (1.1) with logistic source, based on the L^p energy estimates and Moser iteration, Zhang and Li [21] proved the existence of global classical solutions of (1.1) with $\tau = 0, f(u) = \mu u(1-u)$ whenever $\mu > \frac{(N-2)_+}{N}(\chi\alpha - \xi\gamma)$, and also discussed the asymptotic behavior of solutions when $\mu > 2\chi\alpha$. It is also noticed that for (1.1) with $\tau = 0, f(u) \leq a - \mu u^{k+1}$, Li and Xiang [22] proved that the classical solution will exist globally for all $N \geq 2$ in the case $k > 1$. Moreover, when $\xi\gamma = \chi\alpha$ and $k > \frac{\sqrt{N^2+4N}-N}{2}$, the classical solution is uniformly bounded. Additionally, in [23–25] the global boundedness were considered for the attraction–repulsion chemotaxis system with nonlinear diffusion.