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Spectral analysis and global well-posedness for a viscous tropical climate model with only a damp term

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ABSTRACT

Tropical climate model derived by Frierson–Majda–Pauluis in Frierson et al. (2004) has been studied by many works (see, e.g., Li and Titi (2016); Li and Titi (2016); Wan (2016)). In this paper, we consider the Cauchy problem for the viscous tropical climate model with only a damp term and obtain the global well-posedness for the small data. The estimate of $\int_0^t \|\nabla\theta\|^2_{L^\infty}\, d\tau$ is the main difficulty. To achieve our goal, we need using the spectral analysis to the system and constructing a new energy estimate of u in Besov space.

1. Introduction

In this paper, we consider the Cauchy problem for the viscous tropical climate model with only one damp term which reads as follows:

$$\begin{cases} \partial_t u + u\cdot\nabla u - \Delta u + \nabla p = -\mathrm{div}(v\otimes v), \\ \partial_t v + u\cdot\nabla v + v\cdot\nabla u + v = \nabla\theta, \\ \partial_t\theta + u\cdot\nabla\theta = \mathrm{div} v, \\ \mathrm{div} u = 0, \\ (u,v,\theta)(t,x)|_{t=0} = (u_0(x), v_0(x), \theta_0(x)), \end{cases} \tag{1.1}$$

where $(t,x)\in\mathbb{R}^+\times\mathbb{R}^2$, $u=(u^1,u^2)$, $v=(v^1,v^2)$ stand for the barotropic mode and the first baroclinic mode of the vector velocity, respectively, p, θ represent the scalar pressure, scalar temperature, respectively. We call $-\Delta u$ the viscous term or the Laplacian term, while the fourth term v of the left-hand side of $(1.1)_2$ is the damped term. Let us note that u satisfies the divergence free condition.

By performing a Galerkin truncation to the hydrostatic Boussinesq equations, Frierson–Majda–Pauluis in [1] derived the original version of (1.1) without any Laplacian terms, of which the first baroclinic mode had

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been originally used in some studies of tropical atmosphere in [2,3]. For more details on the first baroclinic mode, we refer to Sections 1 and 2 in [1] and references therein.

Li–Titi [4] had proved the global well-posedness for a tropical climate model with two Laplacian terms which can be derived from replacing v in $(1.1)_2$ by $-\Delta v$. In that paper, one can see it is difficult to get the gradient estimate of (u,v,θ) directly due to the absence of thermal diffusion. But via constructing a new unknown $\mathfrak{w}$,

$$\mathfrak{w} :\overset{def}{=} v - \nabla(-\Delta)^{-1}\theta,$$

they improved the regularity of u, and then obtained the gradient estimate of (u,v,θ). It is clear that the Laplacian term $-\Delta u$ plays a key role in their proof. We also refer to a related work [5].

Provided without the Laplacian term $-\Delta v$, it seems difficult to obtain the gradient estimate of (u,v,θ). And adding a damping term v does not appear to be sufficient to overcome this difficulty. To the best of our knowledge, many damped models such as damped Navier–Stokes equations [6], compressible Euler equations [7] and MHD equations [8] have been studied. Due to $(1.1)_3$ without any damping or dissipation, one can see it is a involved problem to establish the global well-posedness for (1.1) even with small initial data. Our aim here is for small global strong solution.

Very recently, the author [9] studied a new model given by

$$\begin{cases} \partial_t u + u\cdot\nabla u + \alpha u + \nabla p = -\mathrm{div}(v\otimes v), \\ \partial_t v + u\cdot\nabla v + v\cdot\nabla u + \alpha v - \eta\Delta v = \nabla\theta, \\ \partial_t\theta + u\cdot\nabla\theta = \mathrm{div} v, \\ \mathrm{div} u = 0, \\ (u(0,x), v(0,x), \theta(0,x)) = (u_0(x), v_0(x), \theta_0(x)) \end{cases}$$

and obtained the global well-posedness with small data. The difficulty is that we need a temperature diffusion to close the estimate of the nonlinear term $u\cdot\nabla\theta$. By a new analysis of the linear key part

$$\begin{cases} \partial_t v + \alpha v - \eta\Delta v - \nabla\theta = 0, \\ \partial_t\theta - \mathrm{div} v = 0 \end{cases}$$

and a new unknown

$$\Omega :\overset{def}{=} \mathcal{R}v + \eta\Lambda\theta,$$

where $\mathcal{R} := \Lambda^{-1}\mathrm{div}$, the hidden temperature diffusion was obtained.

Now, let us show our main result.

Denote

$$E_1(t) := \sup_{0\le\tau\le t}\|(u,v,\theta)(\tau)\|_{H^s}^2 + \int_0^t \|(\nabla u, v)(\tau)\|_{H^s}^2 d\tau,$$

$$E_2(t) := \sup_{0\le\tau\le t}\|u(\tau)\|_{\dot B^0_{2,1}} + \int_0^t \|u(\tau)\|_{\dot B^2_{2,1}} d\tau,$$

and

$$E_1(0) := \|(u_0,v_0,\theta_0)\|_{H^s}^2,\ E_2(0) := \|u_0\|_{\dot B^0_{2,1}}.$$

Theorem 1.1. *Consider (1.1) with the initial data (u_0,v_0,θ_0) satisfying*

$$(u_0,v_0,\theta_0)\in H^s(\mathbb{R}^2),\ s>3 \text{ and } u_0\in \dot B^0_{2,1}(\mathbb{R}^2),\ \mathrm{div} u_0 = 0.$$