Optik 154 (2018) 261–266

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Full length article

Equivalence analysis of retarder compose of three wave plates

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ARTICLE INFO

Article history:
Received 4 July 2017
Accepted 8 October 2017

Keywords:
Equivalence
Retardation
Azimuth
Retarder
Wave plate

ABSTRACT

For light of a given wavelength, an optical system containing several retardation plates is optically equivalent to a system containing only two elements-one is retardation plate, and the other is a rotator. In this paper, by means of Jones matrix, we have studied the system containing three plates, which can work as only one pure retarder without rotator if the conditional equation is satisfied. The governing equations for equivalence is deduced, which determine the overall system retardation and azimuth. The general condition equations are discussed in detail and several particular cases are simulated. This method is instructive and convenient for compound retarders designing.

1. Introduction

High-purity polarized light is essential in numerous experiments and applications [1–4].To obtain polarized light, linear polarizers [5,6], birefringence wave plates [7–10], among others [11], can be used. Wave plates made from most birefringent materials can only be used at a single wavelength, since the phase retardation produced by a single plate is usually much less for red light than for blue light. Achromatic wave retarders are commonly constructed by combining several plates of different thickness or birefringent materials [12–17]. In fact, the topic about the combined optical systems, which are composed of pure retarders, linear polarizers, and rotators, has been discussed by R.C.Jones [18,19], which proved the theorem that an optical system containing any number of retarders and rotators is optically equivalent to a system containing only a retarder and a rotator [18]. Pancharatn discussed the equivalence theorems by means of Poincare sphere [7],and Bhattacharya discussed with Jones matrix [20]. Both of them studied the combination of three birefringent plates, but only the case that the first and the last plates have identical retardations. In this article, we will study the general condition that third cascading retarders can work as one pure retarder (with no rotator) by Jones matrix. The equivalent retardation and azimuth of the compound retarder are deduced in part 2. Practical design simulation is given and illuminated in 3D figures in part 3.

2. Theory

Let us consider the scheme shown in Fig. 1. An optical system M is formed by N wave plates, which are placed in a cascade. We now calculate the retardation of the achromatic combination using the well-known Jones matrix method. In mathematical terms, M is obtained as the product of N wave plates matrices, $\mathbf{C}_i(\delta_i, \theta_i), j = 1, \cdots, N$, which are characterized by

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https://doi.org/10.1016/j.ijleo.2017.10.042
0030-4026/

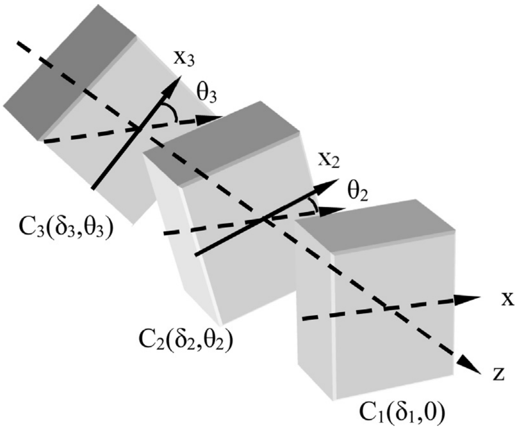

Fig. 1. Scheme of optical system compose of three wave plates of same material placed in a cascaded configuration.

their retardations δ_i and the azimuths of their fast axis θ_i with respect to the reference axis. Without loss of generality, the fast axis of the first plate is defined as reference, i.e. $\theta_1 = 0$; therefore, the angles θ_i represent the angle between the fast axis of the first wave plate and the fast axis of the plate in position i. Suppose that the incidence angle is normal to each plate, the Jones matrix of the optical system is

$$M = \mathbf{C}_N(\delta_N, \theta_N)\cdots\mathbf{C}_1(\delta_1, \theta_1) = \prod_i \mathbf{C}_i(\delta_i, \theta_i) \tag{1}$$

The retardation of each wave plate is defined by

$$\delta_i = \frac{2\pi}{\lambda}(n_e - n_0)d_i \tag{2}$$

where λ represents the incident wavelength, d_i represents the thickness of each wave plate, and the difference Δn = ne −no is the birefringence of the material, ne and no being the extraordinary and ordinary refractive indexes of the material, respectively. The Jones matrix of retarder whose fast axis coincide with the x-axis is given by [10,18].

$$\mathbf{C}(\delta, 0) = \begin{pmatrix} e^{\frac{i\delta}{2}} & 0 \\ 0 & e^{-\frac{i\delta}{2}} \end{pmatrix} \tag{3}$$

where δ is the phase difference introduced by the retarder.

We now calculate the Jones matrix of an oblique retarder whose fast axis makes an angle θwith the x-axis. The corresponding Jones matrix can be deduced from the transformation formula of coordinate frame. If the coordinates frame xoy rotated an angle of θ counterclockwise around the origin O to a new frame x'oy', as shown in Fig. 2, the coordinate projection of incident light vector under the original and final coordinates have the relations:

$$E_{x'} = E_x\cos\theta + E_y\sin\theta, E_{y'} = -E_x\sin\theta + E_y\cos\theta \tag{4}$$

The matrix form is:

$$\begin{pmatrix} E_{x'} \\ E_{y'} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} E_x \\ E_y \end{pmatrix}, or \mathbf{E}_{x',y'} = \mathbf{R}(\theta)\mathbf{E}_{x,y} \tag{5}$$

where

$$\mathbf{R}(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{6}$$

is the rotation matrix of coordinates [10]. As vectors and coordinate systems actually rotate with respect to each other in opposite directions, the rotation matrix is differing slightly from that of reference [14,18,21].

Considering a polarized light beam incident to an oblique retarder in xoy coordinates with azimuth ofθ, as illuminated in Fig. 1. The azimuth lies in x'oy' coordinates, which has an intersection angleθwith the original coordinates frame xoy, as shown in Fig. 2.Assuming the Jones vector of incident light and outgoing are $\mathbf{E}_{in}$ and $\mathbf{E}_{out}$, In the xoy coordinates, we have:

$$\mathbf{E}_{out} = \mathbf{C}(\delta, \theta)\mathbf{E}_{in} \tag{7}$$